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REVIEWS

Stabilization of Nonlinear Dynamic Systems Using the System State Estimates Made by the Asymptotic Observer[1]

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Abstract—Consideration was given to asymptotic stabilization of the equilibria of nonlinear dynamic systems using the dynamic output feedbacks, that is, the feedbacks in the estimate of system state made by the asymptotic observer. Presented were the basic methods of constructing the asymptotic observers for the nonlinear dynamic systems with control and the approaches to system stabilization using the system state estimate made by the observer.

1. INTRODUCTION

Attention paid during the recent decades by many domestic and international researchers (see, for example, [1–131]) to the problems of control of dynamic systems by the output feedbacks is due to the fact that in control of various technical systems the full system state vector is usually unknown and only some functions of the state variables—the system outputs—are observable. As is well known [44], the static output feedbacks enable stabilization under incomplete measured information about the system state only for certain classes of dynamic systems such as systems with passivity or passification of the static output feedback [60, 132–134], and generally fail to provide the desired result.

Extension of the system dynamics owing to the data about the output values provided by the observer (a special dynamic system whose state with time approaches fairly rapidly that of the original system) and use of the dynamic output feedbacks in the form of a function of the observer state and the output of the original system offer an alternative to the static output feedbacks. At that, the observer state at an arbitrary time instant is regarded as an estimate of the system state at this instant. The main problem of observer construction lies in providing the desired dynamics of reduction of the error of system state estimate, that is, the mismatch between the states of system and observer at the same time instant. Asymptotic or exponential error reduction in time is usually desired.

We consider asymptotic stabilization of the equilibrium $x = 0$, $u = 0$ of the dynamic system like

$$\dot{x} = f(x, u), \quad y = h(x), \tag{1}$$

where $x \in R^n$ is the system state vector, $u \in R^m$ is the control, $y \in R^p$ is the measured system output, $f(\cdot, \cdot)$ and $h(\cdot)$ are sufficiently smooth functions of their arguments, and $f(0, 0) = 0$, $h(0) = 0$. Let us assume that the problem of stabilization was solved in the form of the state feedback

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$u = k(x)$, $k(0) = 0$, and the estimate $\widehat{x}$ of system state x provided by the observer is known. Let us consider the control $u = k(\widehat{x})$ obtained from the feedback by replacing the system state by its estimate. The error $e = \widehat{x} - x$ of state estimation may be interpreted as the perturbation on the system through the given control $u = k(\widehat{x}) = k(x + e)$. This gives rise to the question as to whether the so-constructed control in the form of the system state estimate feedback solves the stabilization problem under study.

For the linear system, the affirmative answer has the form of the well-known separation principle [135, 136]. Namely, let the linear state feedback like $u = Kx$, $K \in R^{m\times n}$, which globally asymptotically stabilizes the system equilibrium $x = 0$, $u = 0$ for a known state vector be established for the linear system

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{2}$$

where $x \in R^n$ is the system state vector, $u \in R^m$ is the control, $y \in R^p$ is the measured system output, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, the pair (A, B) is controllable, and the pair (A, C) is detectable [135]. Let us assume that constructed was the asymptotic observer

$$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + Bu,$$

where the gain matrix $L \in R^{n\times p}$ is chosen so that the matrix $A + LC$ has eigenvalues only with negative real parts. For an appropriate feedback $u = K\widehat{x}$ in the estimate $\widehat{x}$ of the state vector x, the system then retains its global asymptotic stability.

For the nonlinear system, the answer is generally affirmative only for the local asymptotic stabilization of the given equilibrium. The paper [1] demonstrated validity of the separation principle in the case of local asymptotic stabilization of the nonlinear weakly detectable systems like (1) which have continuously differentiable function $f(\cdot,\cdot)$ in their right-hand side and are locally asymptotically stabilized by the continuously differentiable state feedback. These findings were extended in [5] to the case where the function $f(\cdot,\cdot)$ in the right-hand side of system (1) is continuous and the stabilizing controls belong to the class of continuous state feedbacks.

For the problem of global asymptotic stabilization of the given equilibrium, the answer is generally negative because there are examples of nonlinear systems to which the separation principle is inapplicable in the case of global stabilization. This may be due to an unlimited increase in the solutions of the system with the control $u = k(\widehat{x})$ in a finite time before the error $e = \widehat{x} - x$ of the system state estimate vanishes [25, 32, 137, 138].

The following key questions concerning the global asymptotic system stabilization by separate construction of the stabilizing state feedback and the observer for estimation of the system state with subsequent substitution of the state estimate in the feedback were formulated for the nonlinear dynamic systems [32]:

(1) Let the control law in the form of a state feedback $u = k(x)$ that globally asymptotically stabilizes the given system equilibrium be established and the global observer providing the estimate $\widehat{x}$ of the system state vector x be constructed. Will the system be globally asymptotically stable for the control $u = k(\widehat{x})$?

(2) Let a global observer be constructed. Is there a control law in the form of the state feedback $u = k(x)$ that globally asymptotically stabilizes the given system equilibrium such that the control $u = k(\widehat{x})$ provides global asymptotic stability of the closed-loop system?

(3) There is a control law in the form of the state feedback $u = k(x)$ that globally asymptotically stabilizes the given system equilibrium. Is there a global observer such that the control $u = k(\widehat{x})$ makes the closed-loop system globally asymptotically stable?

(4) Are there a global observer and a control law as the state feedback $u = k(x)$ globally asymptotically stabilizing the given system equilibrium such that the system is globally asymptotically stable under the control $u = k(\widehat{x})$?

It is known (see, for example, [32, 137, 139]) that for the nonlinear systems the answer to the first question is generally negative even if the error $e = \widehat{x} - x$ of system state estimate made by the observer decreases exponentially with time. According to [32], the answer remains negative even if the state estimate error vanishes in a finite time because under the control $u = k(\widehat{x})$ the system solutions can grow unlimitedly in a finite time before the system state estimate vanishes.

The reader can find in [32, 138] an example of a nonlinear dynamic system for which the answer to the second question also is negative for the class of controls consisting of all continuous state feedbacks like $u = k(x)$. For the given class of feedbacks, this provides a negative answer to the second question in the general case.

Particular examples of systems presented in [25] demonstrate that uniform observability under an arbitrary control [140] and global asymptotic stabilizability of the equilibrium $x = 0$, $u = 0$ of the nonlinear system like (1) by the continuous state feedback like $u = k(x)$, $k(0) = 0$ generally do not secure the global asymptotic stabilizability of the system by a dynamic output feedback given by

$$u = \widetilde{k}(\xi, y), \quad \dot{\xi} = \phi(\xi, y), \tag{3}$$

where $\xi \in R^l$ for some $l > 0$ and $\widetilde{k}(\cdot,\cdot)$ and $\phi(\cdot,\cdot)$ are some continuous functions of their arguments, $\widetilde{k}(0,0) = 0$, $\phi(0,0) = 0$. For example, the equilibrium $x = 0$, $u = 0$ of the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_2^n + u, \quad y = x_1, \tag{4}$$

where $x = (x_1, x_2)^{\mathrm{T}} \in R^2$ is the system state vector, $u \in R$ is the control, $y \in R$ is the measured system output, and $n \in N$, is globally asymptotically stabilizable by the continuous state feedback. Since $x_1 = y$, $x_2 = \dot{y}$, system (4) is uniformly observable under an arbitrary control [11, 24, 140]. According to [25], however, for $n \geq 3$ there exists no continuous dynamic output feedback like (3) that globally asymptotically stabilizes the system $x = 0$, $\xi = 0$ of system (4) with control (3).

The impossibility of stabilizing by the dynamic output feedbacks the equilibria in the aforementioned examples of nonlinear systems is due to the fact that these systems lack "observability of solution unlimitedness" [25]. Availability of this property in a dynamic system implies that if some system solution grows unlimitedly in a finite time, then this function of system output as considered over the given solution also grows over this time. According to [25], the following class of nonlinear dynamic systems

$$\begin{gathered}
\dot{z} = H(z, x), \\
\dot{x}_1 = x_2, \\
\vdots \\
\dot{x}_{r-1} = x_r, \\
\dot{x}_r = x_r^k + F(z, x_1, \ldots, x_{r-1}) + G(z, x_1, \ldots, x_{r-1})u, \\
y = x_1,
\end{gathered} \tag{5}$$

where $x = (x_1, \ldots, x_r)^{\mathrm{T}} \in R^r$, $r > 2$, $z \in R^n$, $u \in R$, $y \in R$, and $H(\cdot)$, $F(\cdot)$, and $G(\cdot)$ are continuous functions of their arguments, $H(0,0) = 0$, $F(0,0) = 0$, $G(z,x) \neq 0$ for all $z \in R^n$ and $x \in R^r$, does not feature "observability of solution unlimitedness" for $k \geq r/(r-1)$. Consequently, for $k \geq r/(r-1)$ the global asymptotic stabilization of the equilibrium $z = 0$, $x = 0$, $\xi = 0$ of system (5) with control (3) by the continuous dynamic output feedback like (3) is impossible. In particular, we note that the results of [25] imply negative answers to the third and fourth questions.

Classes of nonlinear dynamic systems like (1) for which there are affirmative answers (also obtained by the present writers) to some of the above questions about asymptotic stabilization of the given system equilibrium by a separate stabilizing state feedback and an observer estimating the system state with subsequent substitution of the state estimate in the feedback can be found in Section 4.

Direct stabilization of a special extended system consisting of the equations of the original system and the equations of system state estimation by the observer by means of the control laws as the observer state functions and the output of the original system offers a method of asymptotic system equilibrium stabilization by the dynamic output feedbacks which is an alternative to the separation principle. The method of backstepping in observer [9, 137] that is discussed in Section 4.4 marked a substantial progress in this direction.

Notation and definitions used in what follows are presented in Section 2. The main methods of constructing asymptotic global observers for the nonlinear dynamic systems with control as well as the classes of nonlinear systems admitting construction of asymptotic global observers are considered in Section 3.

Part of the results presented in this paper is compiled in Tables 1 and 2 (see Appendix) describing the classes of nonlinear systems admitting construction of the asymptotic observers, conditions for and types of stability of the system describing the dynamics of the error of state estimation of the original system by the observer, conditions for validity and type of the separation principle (in the case of local, global asymptotic stabilization or asymptotic stabilization in the large).

2. NOTATION AND DEFINITIONS

The following notation and definitions are used in the present paper.

Definition 2.1 ([25]). Let $x(t)$ be an arbitrary noncontinuable solution of system (1) determined over the half-interval $[0, T)$, $0 < T < +\infty$, and corresponding to some input function $u(t)$ which is arbitrary, continuous, and bounded on $[0, T)$. If the equality

$$\limsup_{t \to T} |h(x(t))| = +\infty$$

is satisfied, then the dynamic system (1) is said to feature "observability of solution unlimitedness."

Definition 2.2 (see, for example, [137, 139, 141, 142]). The continuous function $\alpha : [0, d) \to R^+$, $R^+ = [0, +\infty)$, $0 < d \leq +\infty$, is called the function of the class K if it is strictly growing and $\alpha(0) = 0$.

The function $\alpha(\cdot)$ of the class K is called the function of the class K_∞ if $d = +\infty$ and $\alpha(s) \to +\infty$ for $s \to +\infty$.

The continuous function $\beta : [0, d) \times R^+ \to R^+$ is called the function of the class KL if for any fixed t the function $\beta(s, t)$ is that of the class K relative to the variable s and for any fixed s $\beta(s, t) \to 0$ for $t \to +\infty$.

Definition 2.3 (see, for example, [5, 14]). The dynamic system (1) is called the weakly detectable system if there are

(1) continuous function $g : R^n \times R^p \times R^m \to R^n$, $g(0, 0, 0) = 0$;

(2) continuously differentiable function $W : R^n \times R^n \to R^+$;

(3) functions $\alpha_i : [0, d) \to R^+$, $i = \overline{1, 3}$, of the class K for some $d > 0$ such that the equality $f(x, u) = g(x, h(x), u)$ is satisfied for all $x \in R^n$, $u \in R^m$ and the following inequalities are valid

for all x, $\widehat{x} \in R^n$, $u \in R^m$, and sufficiently small $e = \widehat{x} - x \in R^n$ satisfying the condition $\mid e \mid < d$:

$$\alpha_1(|\widehat{x} - x|) \le W(x, \widehat{x}) \le \alpha_2(|\widehat{x} - x|), \tag{6}$$

$$\frac{\partial W}{\partial x} f(x, u) + \frac{\partial W}{\partial \widehat{x}} g(\widehat{x}, h(x), u) \le -\alpha_3(|\widehat{x} - x|). \tag{7}$$

The dynamic system (1) is regarded as detectable if (i) the weak detectability conditions are met; (ii) $d = +\infty$ and besides the functions $\alpha_i(\cdot)$, $i = 1, 2$, belong to the class K_∞; (iii) inequalities (6) and (7) are valid for all $x, \widehat{x} \in R^n$, $u \in R^m$ and all $e = \widehat{x} - x \in R^n$.

Here and below, $\mid \cdot \mid$ stands for the Euclidean norm in R^n. We note that in the case where the dynamic system (1) is linear and has the form (2) we use the notion of detectability of the pair (A, C), which is equivalent to existence of the matrix $L \in R^{n \times p}$ such that the eigenvalues of $A + LC$ have only negative real parts [135].

Definition 2.4 ([140]). The dynamic system (1) is called the uniformly observable system under arbitrary control if the initial system state $x(0)$ is uniquely determined from the output trajectory $y(t)$, $t \in [0, T]$, for any continuous bounded input function $u(t)$ defined over an interval $[0, T]$, $T > 0$, such that for arbitrary initial values $x(0)$ the solutions $x(t)$ of the system are defined over this interval.

We note that various necessary and sufficient conditions for uniform observability under arbitrary control can be found in [11, 24, 88, 140, 143–147] for the nonlinear dynamic systems like (1).

Definition 2.5 ([148, 149]). If there are functions $\beta(\cdot, \cdot)$ and $\gamma(\cdot)$ of the respective classes KL and K (for $d = +\infty$) such that for any $x(0)$ and an arbitrary continuous input function $u(t)$ bounded over $[0, +\infty)$ the solutions $x(t)$ of system (1) exist for all $t \ge 0$ and satisfy the inequality

$$|x(t)| \le \beta(|x(0)|, t) + \gamma\left(\sup_{0 \le \tau \le t} |u(\tau)|\right)$$

for all $t \ge 0$, then the dynamic system (1) is said to have stable input-state map relative to the input u.

We recall that $f(x, u)$, $x \in R^n$, $u \in R^m$, $f : R^n \times R^m \to R^n$, is global Lipschitzian in x uniformly in u with the constant γ_f if

$$|f(x_1, u) - f(x_2, u)| \le \gamma_f |x_1 - x_2| \quad \text{for any} \quad x_1, x_2 \in R^n, \quad u \in R^m.$$

The paper makes use of $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ to denote, respectively, the magnitude-maximal and magnitude-minimal eigenvalues of an arbitrary matrix P. The Jacoby matrix of the function $f(x, u)$, $x \in R^n$, $u \in R^m$, $f : R^n \times R^m \to R^n$ in part of the variables x is denoted by $D_x f(x, u)$. By the matrix inequality $A \le B$, where A, B are arbitrary symmetrical $n \times n$ matrices, is meant the inequality $x^{\mathrm{T}} A x \le x^{\mathrm{T}} B x$ for all $x \in R^n$.

In what follows, by the problem of stabilization of a dynamic system like (1), is meant asymptotic stabilization of the given system equilibrium $x = 0$, $u = 0$. By the class of permissible control laws is meant the class of continuous functions of the form $u = u(t, x, y)$ or $u = u(t, \widehat{x}, y)$, where $x \in R^n$ is the system state vector, $\widehat{x} \in R^n$ is the vector of system estimation by the observer, and $y \in R^p$ is the measured system output. The possible dependence on time is due here to the permissible presence of external signals in the control laws, for example, to the dependence on the solutions of other dynamic systems.

3. ASYMPTOTIC OBSERVERS FOR THE NONLINEAR DYNAMIC SYSTEMS WITH CONTROL

There exist several basic approaches to constructing asymptotic observers for the nonlinear dynamic systems with control. One of the well-known methods of constructing the asymptotic local observer for a nonlinear system is based on system linearization in the neighborhood of a given point and constructing an observer for the resulting linear system. However, the convergence domain of this observer usually is small, and its estimation is difficult. In this connection, of great interest are the methods of constructing observers operating globally or in the large. Various methods of constructing observers for the nonlinear dynamic systems which operate globally or in the large can be found, for example, in [83, 89–91, 144–146, 150–170].

3.1. Let us consider the case of the affine nonlinear dynamic system (1) given by

$$\dot{x} = A(x) + \sum_{j=1}^{m} B_j(x)u_j, \quad y = h(x), \tag{8}$$

where $x \in R^n$ is the system state vector; $A(x)$ and $B_j(x)$, $j = \overline{1,m}$, are the smooth vector fields on R^n; $y \in R^p$ is the measured system output, $h(x) = (h_1(x), \dots, h_p(x))^{\mathrm{T}}$, $h_i(x) \in C^\infty(R^n)$, and $i = \overline{1,p}$; $u = (u_1, \dots, u_m)^{\mathrm{T}} \in R^m$ is the vector control. The so-called geometrical method of observer construction is based on rearranging system (8) in a special canonical form [150] which, according to [151], is representable as

$$\begin{aligned} \dot{\chi} &= \begin{pmatrix} A^1 & 0 & \dots & 0 \\ 0 & A^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A^p \end{pmatrix} \chi + \begin{pmatrix} \psi_1(\chi^1_{n_1}, \dots, \chi^p_{n_p}) \\ \psi_2(\chi^1_{n_1}, \dots, \chi^p_{n_p}) \\ \vdots \\ \psi_n(\chi^1_{n_1}, \dots, \chi^p_{n_p}) \end{pmatrix} + \sum_{j=1}^{m} \begin{pmatrix} \widetilde{b}_{1j}(\chi^1_{n_1}, \dots, \chi^p_{n_p}) \\ \widetilde{b}_{2j}(\chi^1_{n_1}, \dots, \chi^p_{n_p}) \\ \vdots \\ \widetilde{b}_{nj}(\chi^1_{n_1}, \dots, \chi^p_{n_p}) \end{pmatrix} u_j \\ &= A\chi + \psi\left(\chi^1_{n_1}, \dots, \chi^p_{n_p}\right) + \sum_{j=1}^{m} \widetilde{B}_j\left(\chi^1_{n_1}, \dots, \chi^p_{n_p}\right) u_j, \quad y = H\left(\chi^1_{n_1}, \dots, \chi^p_{n_p}\right), \end{aligned} \tag{9}$$

where $n = n_1 + \ldots + n_p$ is the observability multiindex [151]; $\chi^k = (\chi^k_1, \dots, \chi^k_{n_k})^{\mathrm{T}}$, $k = \overline{1,p}$, are the n_k-dimensional vectors; $\chi = \left((\chi^1)^{\mathrm{T}}, \dots, (\chi^p)^{\mathrm{T}}\right)$; $A^k = (a^k_{ij})$, $i = \overline{1,n_k}$, $j = \overline{1,n_k}$, $k = \overline{1,p}$, are the n_k-order square matrices with the elements $a^k_{ij} = 1$, if $i - j = 1$, and $a^k_{ij} = 0$, if $i - j \neq 1$. The existence conditions for the change of variables rearranging system (8) in (9) are given in [150, 151].

If for the function $y = H(\chi^1_{n_1}, \dots, \chi^p_{n_p})$ there exists a feedback function $(\chi^1_{n_1}, \dots, \chi^p_{n_p})^{\mathrm{T}} = H^{-1}(y)$, then the asymptotic observer for system (9) is represented by the dynamic system

$$\begin{gathered} \dot{\widehat{\chi}} = A\widehat{\chi} + LC(\widehat{\chi} - \chi) + \psi(\chi^1_{n_1}, \dots, \chi^p_{n_p}) + \sum_{j=1}^{m} \widetilde{B}_j(\chi^1_{n_1}, \dots, \chi^p_{n_p})u_j, \\ (\chi^1_{n_1}, \dots, \chi^p_{n_p})^{\mathrm{T}} = H^{-1}(y), \end{gathered} \tag{10}$$

where $C = (C_{ij})$, $i = \overline{1,p}$, $j = \overline{1,p}$, is the block matrix whose elements C_{ij}, $i = \overline{1,p}$, $j = \overline{1,p}$, are row matrices of the length n_j and at that $C_{ij} = (0, \dots, 0, 1)$, if $i = j$, and $C_{ij} = (0, \dots, 0, 0)$, if $i \neq j$. In observer (10), the $L \in R^{n \times p}$ gain matrix defines dynamics of the error $e = \widehat{\chi} - \chi$ of state estimation of system (9). In systems (9) and (10), consideration is given to an identical control which belongs to the permissible class of control laws and is such that the solutions $\chi(t)$ of system (9) with the given control are determined for all $t \geq 0$.

Subsequent constructions in this case may be based on the linear technique providing exponential decrease of the error at least in the canonical coordinates. Indeed, the equation $e = \widehat{\chi} - \chi$ of the error of estimation by the observer (10) of the state of system (9) for identical control in systems (9) and (10) has the form

$$\dot{e} = (A + LC)e, \tag{11}$$

where the observer gain matrix L is chosen so that the matrix $A + LC$ has eigenvalues with negative real parts only. This choice of L is possible because the pair (A, C) is observable [135]. Consequently, the equilibrium $e = 0$ of system (11) is exponentially stable and the state estimation error is control-independent and exponentially tends to zero.

The geometrical method of constructing the observer for the nonlinear dynamic systems is discussed in detail, for example, in [150–155]. We note that the results become global if the corresponding changes of variables are defined globally, the map $H : R^p \to R^p$ is invertible, and for the control u under consideration from the permissible class of control laws, any solution $\chi(t)$ of (9) is defined for all $t \geq 0$.

3.2. A similar approach can be developed for the non-affine nonlinear dynamic systems like (1). Let us assume that system (1) has the form

$$\dot{x} = Ax + \varrho(y, u), \quad y = Cx, \tag{12}$$

where $x \in R^n$; $A \in R^{n\times n}$ and $C \in R^{p\times n}$ are the constant matrices, the pair (A, C) is detectable, and the map $\varrho : R^p \times R^m \to R^n$ is continuous. For system (12), the global asymptotic observer is as follows:

$$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + \varrho(y, u). \tag{13}$$

The gain matrix $L \in R^{n\times p}$ determines dynamics of the error $e = \widehat{x} - x$ of state estimation of system (12). In systems (12) and (13), consideration is given to the identical controls which belong to the permissible class of control laws and are such that for any $x(0)$ the solutions $x(t)$ of system (12) with the given control are determined for all $t \geq 0$.

Under identical controls in systems (12) and (13), the equation of the error $e = \widehat{x} - x$ of estimation by observer (13) of the state of system (12) has the form (11), where the observer gain matrix L is taken so that the eigenvalues of the matrix $A + LC$ have only negative real parts. In this case, the equilibrium $e = 0$ of system (11) with the right-hand side under consideration is globally exponentially stable and the error of estimation the state of system (12) by observer (13) is control-independent and tends exponentially to zero.

3.3. Another procedure [156–158] enables one to construct an asymptotic global observer for the following nonlinear dynamic system (1):

$$\dot{x} = Ax + \widetilde{f}(x, u) + \varrho(y, u), \quad y = Cx, \tag{14}$$

where $x \in R^n$ is the system state vector; $A \in R^{n\times n}$ and $C \in R^{p\times n}$ are constant matrices, the pair (A, C) is detectable; $y \in R^p$ is the measured system output; $u \in R^m$ is the control; the maps $\varrho : R^p \times R^m \to R^n$ and $\widetilde{f} : R^n \times R^m \to R^n$ are continuous, the function $\widetilde{f}(x, u)$ being global Lipschitzian in x uniformly in u with constant $\gamma_{\widetilde{f}}$.

For system (14), the observer is given by

$$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + \widetilde{f}(\widehat{x}, u) + \varrho(y, u), \tag{15}$$

where $L \in R^{n\times p}$ is the observer gain matrix to be determined. The equation of the error $e = \widehat{x} - x$ of estimating the state of system (14) by observer (15) is given by

$$\dot{e} = (A + LC)e + \left[\widetilde{f}(x + e, u) - \widetilde{f}(x, u)\right]. \tag{16}$$

The procedure of observer construction comes to seeking a gain matrix L for which the equilibrium $e = 0$ of system (16) is globally asymptotically stable for an arbitrary fixed control u from the permissible class of control laws and any solution $x(t)$ of system (14) with the given control. The following theorem is known.

Theorem 3.1 ([156]). *Let us assume that any solution $x(t)$ of system* (14) *is determined for all $t \geq 0$ under some fixed control u from the permissible class of control laws. Let the gain matrix L in observer* (15) *be taken so as to satisfy the inequality $\gamma_{\widetilde{f}} < \lambda_{\min}(Q)/(2\lambda_{\max}(P))$, where P and Q are positive definite symmetrical matrices satisfying the Lyapunov equation*

$$(A + LC)^{\mathrm{T}} P + P(A + LC) = -Q. \tag{17}$$

Then, for the given control u and an arbitrary solution $x(t)$ of system (14) *with this control, the equilibrium $e = 0$ of system* (16) *is globally exponentially stable.*

Theorem 3.1 is proved in the Appendix.

We note that Theorem 3.1 allows one only to check for stability system (16) with a particular gain matrix L and gives no answer to the question as to how to determine L because (i) there exists no explicit dependence between the eigenvalues of the matrix $A + LC$ and $\lambda_{\max}(P)$ and (ii) variations in $\lambda_{\max}(P)$ need not be related with the variations of the eigenvalues of $A + LC$ [157].

Sufficiently constructive necessary and sufficient conditions for stability of the equilibrium $e = 0$ of system (16) were obtained in [158]. This publication presented conditions to be satisfied by the matrix $A + LC$. Numerical algorithms to determine the gain matrix L of observer (15) which makes system (16) stable are described in [157, 158]. Construction of an asymptotic observer for the nonstationary system (14) was discussed in [159].

3.4. Let us consider the case where system (14) has the lower triangular form

$$\dot{x} = Ax + a(x) + b(x, u) + \varrho(y, u), \quad y = Cx, \tag{18}$$

where $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in R^n$ is the system state vector, $u \in R^m$ is the control, $y \in R$ is the measured system output; $A = (a_{ij})$, $i = \overline{1, n}$, $j = \overline{1, n}$, is the n-order square matrix with the elements $a_{ij} = 1$, if $j - i = 1$, and $a_{ij} = 0$, if $j - i \neq 1$; $C = (1, 0, \ldots, 0)$ is the row vector of length n; $b(x, u) = (b_1(x_1, u), \ldots, b_i(x_1, \ldots, x_i, u), \ldots, b_n(x, u))^{\mathrm{T}}$, the vector function $a(x) = (0, \ldots, 0, a_n(x))^{\mathrm{T}}$ is an n-dimensional vector, the function $a_n(x)$ being global Lipschitzian and the functions $b_i(x, u)$, $i = \overline{1, n}$, continuous and global Lipschitzian in x uniformly in u; the map $\varrho : R \times R^m \to R^n$ is continuous.

Existence of a change of variables rearranging the dynamic system like (1) with a scalar output in a lower triangular form (18) is a sufficient condition for its uniform observability under an arbitrary control. As was shown in [144–146], a global asymptotic observer for system (18) always exists and is as follows:

$$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + a(\widehat{x}) + b(\widehat{x}, u) + \varrho(y, u), \tag{19}$$

where $L = (l_1, \ldots, l_n)^{\mathrm{T}} \in R^n$ is the observer gain vector to be determined.

The error equation $e = \widehat{x} - x$ of the system (18) state estimate by observer (19) is as follows:

$$\dot{e} = (A + LC)e + G(e, x, u), \tag{20}$$

where

$$G(e, x, u) = (g_1(e, x, u), \ldots, g_n(e, x, u))^{\mathrm{T}},$$
$$g_i(e, x, u) = b_i(x_1 + e_1, \ldots, x_i + e_i, u) - b_i(x_1, \ldots, x_i, u), \quad i = \overline{1, n-1},$$
$$g_n(e, x, u) = b_n(x + e, u) - b_n(x, u) + a_n(x + e) - a_n(x).$$

The results of [144–146] underlie the following statement.

Theorem 3.2. *Let us assume that any solution $x(t)$ of system* (18) *be determined for all $t \geq 0$ under some fixed control u from a permissible class of control laws. Then, for the gain vector L in observer* (19) *such that $l_i = \theta^i k_i$, $i = \overline{1,n}$, where k_i, $i = \overline{1,n}$, are the components of the vector $K = (k_1, \ldots, k_n)^{\mathrm{T}}$ such that the matrix $A_C = A + KC$ has eigenvalues with negative real parts only, there exists a value $\theta_* \geq 1$ of the parameter θ such that for all $\theta > \theta_*$ the equilibrium $e = 0$ of system* (20) *is globally exponentially stable under the given control u and an arbitrary solution $x(t)$ of system* (18) *with the given control.*

Theorem 3.2 is proved in the Appendix.

In [144–146] a matrix method of defining the gain vector for observer (19) was proposed in the form of $L = -P^{-1}C^{\mathrm{T}}$, where the matrix $P = P^{\mathrm{T}} > 0$ is the solution of the equation

$$A^{\mathrm{T}}P + PA + \theta P - C^{\mathrm{T}}C = 0 \tag{21}$$

for an arbitrary and sufficiently great fixed parameter $\theta > 1$ which, as was established in these publications, exists and is unique. For this choice of the vector L, observer (19) traditionally is called the high-gain observer. The following result is presented without proof.

Theorem 3.3 ([144–146]). *Let us assume that under some fixed control u from a permissible class of control laws any solution $x(t)$ of system* (18) *is definite for all $t \geq 0$. Then, upon choosing the gain vector $L = -P^{-1}C^{\mathrm{T}}$ in observer* (19) *where the matrix $P = P^{\mathrm{T}} > 0$ is a unique positive definite solution of Eq.* (21) *there exists a value $\theta_* \geq 1$ of the parameter θ such that, under the given control u and arbitrary solution $x(t)$ of system* (18) *with the given control, for all $\theta > \theta_*$ and any initial values $e(0)$ the solutions $e(t)$ of system* (20) *are definite for all $t \geq 0$ and for all $t \geq 0$ satisfy the inequality $|e(t)| \leq K(\theta)|e(0)| \exp\left(-\frac{\theta}{4}t\right)$, where $K(\theta)$ is a function of the parameter θ such that $K(\theta)\exp\left(-\frac{\theta}{4}\tau\right) \to 0$ is satisfied for an arbitrary fixed $\tau > 0$ if $\theta \to \infty$.*

We note that the methods of defining the gain vector in observer (19) as presented in Theorems 3.2 and 3.3 are directly related. Let the matrix $P_0 = P_0^{\mathrm{T}} > 0$ be a solution of Eq. (21) for $\theta = 1$. Then, the matrix $A_C = A - P_0^{-1}C^{\mathrm{T}}C$ has eigenvalues only with negative real parts because $P_0A_C + A_C^{\mathrm{T}}P_0 = -P_0 - C^{\mathrm{T}}C \leq -P_0$. Now, we take into consideration that $\Lambda A \Lambda^{-1} = \theta A$ and $C\Lambda^{-1} = C$, where $\Lambda = \mathrm{diag}\,(1, 1/\theta, \ldots, 1/\theta^{n-1})$, and demonstrate that the solution of Eq. (21) for any fixed $\theta > 1$ has the form $P = \frac{1}{\theta}\Lambda P_0 \Lambda$ and the choice of the gain vector $L = (l_1, \ldots, l_n)^{\mathrm{T}} = -P^{-1}C^{\mathrm{T}}$ in observer (19) means that $l_i = \theta^i k_i$, $i = \overline{1,n}$, where k_i, $i = \overline{1,n}$, are the components of the vector $K = -P_0^{-1}C^{\mathrm{T}}$ [146]. The same book considered construction of high-gain observers for more general systems including those with vector output.

3.5. The publications [83, 89–91, 160] considered construction of the global asymptotic observers for the nonlinear dynamic systems with the control given by

$$\dot{x} = Ax + G\psi(Hx) + \varrho(y, u), \quad y = Cx, \tag{22}$$

where $x \in R^n$ is the system state vector; $A \in R^{n\times n}$, $G \in R^{n\times r}$, $H \in R^{r\times n}$, and $C \in R^{p\times n}$ are constant matrices, the pair (A, C) is detectable; $y \in R^p$ is the measured system output; $u \in R^m$ is the control; and the maps $\varrho : R^p \times R^m \to R^n$ and $\psi : R^n \to R^r$ are continuous. Here, the nonlinear vector function $\psi(Hx)$ is an r-dimensional vector whose components ψ_i are functions of a linear combination of the state variables

$$\psi_i = \psi_i\left(\sum_{j=1}^{n} H_{ij}x_j\right), \quad i = \overline{1,r},$$

and satisfy each the inequality

$$a \leq \frac{\psi_i(z_1) - \psi_i(z_2)}{z_1 - z_2} \leq b, \quad \forall z_1, z_2 \in R, \quad z_1 \neq z_2. \tag{23}$$

For $a = 0$, $b = \infty$, relations (23) define nondecreasing functions, and for $-a = b = \gamma$, global Lipschitzian functions.

For system (22), the observer is as follows:

$$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + G\psi(H\widehat{x} + K(C\widehat{x} - y)) + \varrho(y, u), \tag{24}$$

where $L \in R^{n\times p}$ and $K \in R^{r\times p}$ are the observer gain matrices to be determined. The equation of error $e = \widehat{x} - x$ of estimation by observer (24) of the state of system (22) is as follows:

$$\dot{e} = (A + LC)e + G[\psi(w) - \psi(v)], \tag{25}$$

where $v = Hx$, $w = H\widehat{x} + K(C\widehat{x} - y)$.

Let us assume that the scalar components of the vector function $\psi(\cdot)$ satisfy inequalities (23) with $a = 0$. If $a \neq 0$, then one can always define a new function $\widetilde{\psi}(v) = (\widetilde{\psi}_1(v_1), \ldots, \widetilde{\psi}_r(v_r))^{\mathrm{T}}$ whose coordinate functions $\widetilde{\psi}_i(v_i) = \psi_i(v_i) - av_i$, $i = \overline{1, r}$, satisfy inequalities (23) with $\widetilde{a} = 0$, $\widetilde{b} = b - a$ and represent system (22) as $\dot{x} = \widetilde{A}x + G\widetilde{\psi}(Hx) + \varrho(y, u)$, $y = Cx$, where $\widetilde{A} = A + aGH$.

It follows from inequalities (23) that

$$\psi_i(w_i) - \psi_i(v_i) = \delta_i(t)(w_i - v_i), \quad i = \overline{1, r},$$

where $\delta_i(t)$, $i = \overline{1, r}$, are scalar time functions assuming values over the interval $[0, b]$. Consequently, the difference $\psi(w) - \psi(v)$ is representable as

$$\psi(w) - \psi(v) = \Delta(t)(w - v),$$

where $\Delta(t) = \mathrm{diag}\ (\delta_1(t),\ \ldots,\ \delta_r(t))$. With regard for the notation $\eta = w - v$, system (25) is then representable as

$$\dot{e} = (A + LC)e + G\Delta(t)\eta, \quad \eta = (H + KC)e. \tag{26}$$

Construction of the observer lies in determining the gain matrices K and L for which the equilibrium $e = 0$ of the linear nonstationary system (26) is globally asymptotically stable.

Theorem 3.4 ([89, 90]). *Let us assume that under some fixed control u from the permissible class of control laws any solution $x(t)$ of system (22) is definite for all $t \geq 0$. If there is a matrix $P = P^{\mathrm{T}} > 0$ and a constant $\nu > 0$ such that the linear matrix inequality*

$$\begin{bmatrix} (A + LC)^{\mathrm{T}}P + P(A + LC) + \nu I & PG + (H + KC)^{\mathrm{T}} \\ G^{\mathrm{T}}P + (H + KC) & D \end{bmatrix} \leq O, \tag{27}$$

where O is the $(n+r)\times(n+r)$ null matrix, I is the $n\times n$ identity matrix, and $D = \mathrm{diag}\ (-2/b, \ldots, -2/b) \in R^{r\times r}$, is satisfied, then the equilibrium $e = 0$ of system (26) is globally exponentially stable under an arbitrary solution $x(t)$ of system (22) with the given control.

Theorem 3.4 is proved in the Appendix.

Therefore, solution of the problem of estimating the state of system (22) by observer (24) comes to determining the gain matrices K and L such that inequality (27) is satisfied for some $P = P^{\mathrm{T}} > 0$ and $\nu > 0$. Relations (27) represent a linear matrix inequality in ν, P, K, and PL, its necessary and sufficient solvability conditions were obtained in [89, 90, 171].

3.6. Let us consider the method [161, 162] of constructing the global asymptotic observer for the nonlinear dynamic system like (1) with continuously differentiable right-hand side and linear output $y = Cx$, where C is an arbitrary constant $p \times n$ matrix. The system observer is sought in the form

$$\dot{\widehat{x}} = f(\widehat{x}, u) + L(u)(C\widehat{x} - y), \tag{28}$$

where $L : R^m \to R^n$ is the continuous function to be determined. The following assumptions are made about the right-hand side of the system.

Assumption 3.1. *There exists an $n \times n$ matrix $P = P^{\mathrm{T}} > 0$ and a constant $k_1 > 0$ such that for any $z \in \ker C \backslash \{0\}$ there exists a neighborhood S_z such that the inequality*

$$v^{\mathrm{T}} P D_x f(x, u) v \leq -k_1 |\ v\ |^2$$

is satisfied for all $(x, v, u) \in R^n \times S_z \times R^m$.

Assumption 3.2. *There exist a continuous function $p : R^m \to R$ and constant $k_2 > 0$ such that $p(u) \geq k_2$ for any $u \in R^m$ and the inequality*

$$|v^{\mathrm{T}} P D_x f(x, u) v| \leq p(u) |\ v\ |^2$$

is satisfied for all $(x, v, u) \in R^n \times R^n \times R^m$.

The equation of error $e = \widehat{x} - x$ of estimation by observer (28) of the state of system (1) with the right-hand side under consideration and output is as follows:

$$\dot{e} = f(x + e, u) - f(x, u) + L(u)Ce. \tag{29}$$

Observer design lies in seeking a continuous function $L(u)$ for which the equilibrium $e = 0$ of system (29) is globally asymptotically stable for an arbitrary fixed control u from the permissible class of control laws and for the solution $x(t)$ of system (1) with the given control. The following result is presented without a proof.

Theorem 3.5 ([161]). *Let Assumptions 3.1 and 3.2 be satisfied and for all $t \geq 0$ any solution $x(t)$ of system* (1) *with the right-hand side at issue and the linear output be defined for some fixed control u from the permissible class of control laws. Then, if the function $L(u) = -\theta p(u) P^{-1} C^{\mathrm{T}}$ is selected in observer* (28), *then there exists a value $\theta_* > 0$ of the parameter θ such that for all $\theta > \theta_*$ the equilibrium $e = 0$ of system* (29) *is globally exponentially stable for the given control u and arbitrary solution $x(t)$ of system* (1) *with the given control.*

We note that if Assumption 3.1 is satisfied, then the inequality $e^{\mathrm{T}} P D_x f(0, 0) e \leq -k_1 |\ e\ |^2$ is valid for all $e \in \ker\ C$, which, as was shown in [161], is equivalent to detectability of the pair $(D_x f(0, 0), C)$. The results formulated as Theorem 3.5 were extended in [162] to construction of the asymptotic observers for the dynamic systems like (1) with the continuous right-hand side and a nonlinear function at the system output.

3.7. Some recent works (see, for example, [15, 21, 42, 163–166]) consider construction of the asymptotic observer for the dynamic system as a problem of using the output feedback for asymptotic stabilization of the system describing the dynamics of error of estimation of the state of the original system. This problem is solved using system passivity and/or the methods of system stabilization similar to the so-called backstepping or back detour of the integrator [137]. For example, in [165] the problem of designing the asymptotic observer for a dynamic system is considered as that of passification by the static output feedback in part of the variables of a special extended system consisting of the equations of the state estimation error of the observed system and the equations of the system itself. At that, the notion of uniform passivity in part of variables is used [165].

As was shown in [165], this approach generalizes the aforementioned methods of constructing the asymptotic observers for the dynamic systems and enables one to obtain a unique procedure of constructing observers for estimation of the state of the dynamic system.

4. OBSERVER-BASED STABILIZATION OF THE NONLINEAR DYNAMIC SYSTEMS

4.1. Feasibility of local stabilization of the nonlinear systems like (1) by separate construction of the stabilizing state feedback and the asymptotic observer for estimation of the system state with subsequent substitution of the state estimate in the feedback was demonstrated in [1, 5]. The following result is valid.

Theorem 4.1 ([1]). *Let the function $f(x,u)$ in the right-hand side of system* (1) *be continuously differentiable in some neighborhood of the point $(0,0) \in R^n \times R^m$. We assume that system* (1) *is weakly detectable and there exists a continuously differentiable state feedback $u = k(x)$, $k(0) = 0$, which locally asymptotically stabilizes the equilibrium $x = 0$, $u = 0$ of system* (1)*. Then, for the control $u = k(\widehat{x})$, the system*

$$\dot{x} = f(x,u), \quad \dot{\widehat{x}} = g(\widehat{x}, h(x), u), \tag{30}$$

where $g : R^n \times R^p \times R^m \to R^n$ from Definition 2.3*, is locally asymptotically stable at the point $x = 0$, $\widehat{x} = 0$.*

We note that (weak) detectability of system (1) means (see, for example, [162]) that the system $\dot{\widehat{x}} = g(\widehat{x}, h(x), u)$, where $g : R^n \times R^p \times R^m \to R^n$ from Definition 2.3 of (weak) detectability, is the global (local) asymptotic observer of system (1), that is, for an arbitrary fixed control u from the permissible class of control laws such that any solution $x(t)$, $\widehat{x}(t)$ of system (30) with the given control is defined for all $t \geq 0$, $t \to +\infty$, we get $e(t) = \widehat{x}(t) - x(t) \to 0$ for any initial values $x(0)$, $\widehat{x}(0)$ (and sufficiently small $e(0) = \widehat{x}(0) - x(0)$). Indeed, for the system

$$\begin{aligned} \dot{e} &= g(x(t) + e, h(x(t)), u) - f(x(t), u) \\ &= F(x(t), e, u), \quad F(x(t), 0, u) = 0, \quad \forall u \in R^m, \quad \forall t \geq 0, \end{aligned} \tag{31}$$

describing the dynamics of state estimation error $e = \widehat{x} - x$ of system (1), let us consider the function $\widetilde{W}(t,e) = W(x(t), x(t) + e)$, where $W(\cdot,\cdot)$ from Definition 2.3 of (weak) detectability satisfies the inequalities (6) and (7), as the Lyapunov function. The control in system (31) is taken from the permissible class of control laws so that the solutions $x(t)$ of system (1) with the given control are determined for all $t \geq 0$. With regard for the inequality (7) and also the relations $\widehat{x} = x + e$, $\partial\widetilde{W}/\partial e = \partial W/\partial\widehat{x}$ and $\partial\widetilde{W}/\partial t = \partial W/\partial x\, \dot{x}(t) + \partial W/\partial\widehat{x}\, \dot{x}(t)$, the time derivative of the function $\widetilde{W}(t,e)$ along the trajectories of the system (31) for all $e \in R^n$ ($|\, e \,| < d$) and all $t \geq 0$ is given by

$$\begin{aligned} \dot{\widetilde{W}}(t,e)|_{(31)} &= \frac{\partial\widetilde{W}}{\partial t} + \frac{\partial\widetilde{W}}{\partial e} F(x(t), e, u) = \frac{\partial W}{\partial x} f(x(t), u) + \frac{\partial W}{\partial\widehat{x}} f(x(t), u) \\ &+ \frac{\partial W}{\partial\widehat{x}} g(x(t) + e, h(x(t)), u) - \frac{\partial W}{\partial\widehat{x}} f(x(t), u) \leq -\alpha_3(|\, e \,|), \end{aligned} \tag{32}$$

where $\alpha_3(\cdot)$ is a function from the class K of Definition 2.3. We note further that for all $e \in R^n$ ($|\, e \,| < d$) and all $t \geq 0$ the function $\widetilde{W}(t,e)$ satisfies the inequality

$$\alpha_1(|\, e \,|) \leq \widetilde{W}(t,e) \leq \alpha_2(|\, e \,|), \tag{33}$$

where $\alpha_i(\cdot)$, $i = 1, 2$, are functions from the class K_∞ (class K) of Definition 2.3. Then, according to [137, 139], existence of the function $\widetilde{W}(t,e)$ satisfying inequalities (32) and (33) implies t-uniform global (local) asymptotic stability of the equilibrium $e = 0$ of system (31) for an arbitrary fixed control u from the permissible class of control laws such that any solution $x(t)$ of system (1) with the given control is determined for all $t \geq 0$ and for an arbitrary solution $x(t)$ of system (1) with the given control.

The facts (formulated as Theorem 4.1) about local stabilization of the nonlinear dynamic systems using the separation principle were extended in [5] to the case where the function $f(\cdot,\cdot)$ in the right-hand side of system (1) is continuous and the stabilizing controls belong to the class of continuous state feedbacks. Theorem 4.1 was shown [5] to retain its validity not only in the case of continuous differentiability, but in the case of continuity of the corresponding functions as well.

4.2. Stabilization in the large of systems like (1), by the dynamic output feedbacks was considered, for example, in [11, 12, 18, 24, 27, 29, 35, 48, 50, 54, 62, 63, 77, 86, 87, 103]. As was demonstrated in [24], it follows from the global asymptotic stabilizability of the equilibrium $x = 0$, $u = 0$ by a smooth state feedback and the uniform observability for an arbitrary control of system (1) with the scalar input and output and the smooth functions $f(\cdot,\cdot)$ and $h(\cdot)$ in the right-hand side of the system that for any compactum $K \subset R^n$ including the point $x = 0$ there exist a dynamic output feedback of the form (3) for $l = n$ and a compactum $K_\xi \subset R^n$ including the point $\xi = 0$ such that the equilibrium $x = 0$, $\xi = 0$ of system (1) closed by the given dynamic output feedback is asymptotically stable with the attraction domain including $K \times K_\xi$. The present paper uses a high-gain observer operating in the large as the asymptotic observer.

The works [12, 18, 62, 63, 139] solve the problem of stabilization in the large of the nonlinear dynamic systems by means of bounded state feedbacks and high-gain observers with subsequent substitution of the observer's system state estimate in the feedback. When the system state in the stabilizing feedback is replaced by its estimate, this bounded control-based approach enables one to avoid a possible dramatic increase in the transient amplitude of the closed-loop system which drives (see, for example, [12]) the solution of the closed-loop system outside the attraction domain and causes loss of the equilibrium stability. The publications [62, 63, 139] considered stabilization in the large of systems like

$$\begin{aligned} &\dot{x} = Ax + B\phi(x, z, u), \\ &\dot{z} = \psi(x, z, u), \\ &y = Cx,\ \zeta = q(x, z), \end{aligned} \tag{34}$$

where $x \in X \subseteq R^r$ and $z \in Z \subseteq R^l$ make up the system state vector; $y \in Y \subseteq R^p$ and $\zeta \in R^s$ are the measured system outputs: $u \in U \subseteq R^m$ is the control; and $\phi : X \times Z \times U \to R^p$ and $\psi : X \times Z \times U \to R^l$ are the locally Lipschitzian maps with $\phi(0,0,0) = 0$, $\psi(0,0,0) = 0$, $q(0,0) = 0$. In system (34), $A \in R^{r\times r}$, $B \in R^{r\times p}$, and $C \in R^{p\times r}$ are the respective block-diagonal matrices $A = \operatorname{diag}(A^1,\dots,A^p)$, $B = \operatorname{diag}(B^1,\dots,B^p)$, $C = \operatorname{diag}(C^1,\dots,C^p)$. Here, $A^k = (a^k_{ij})$, $i = \overline{1,r_k}$, $j = \overline{1,r_k}$, $k = \overline{1,p}$, are the square r_k-order matrices with the elements $a^k_{ij} = 1$, if $j - i = 1$, and $a^k_{ij} = 0$, if $j - i \neq 1$, $B^k = (0,\dots,0,1)^{\mathrm{T}}$, $k = \overline{1,p}$, are the column vectors of height r_k, and $C^k = (1,0,\dots,0)$, $k = \overline{1,p}$, are the row matrices of length r_k, $r = r_1 + \ldots + r_p$.

It is assumed that there exists a control law in the form of the dynamic feedback

$$u = \widetilde{k}(\vartheta, x, \zeta), \quad \dot{\vartheta} = \Gamma(\vartheta, x, \zeta), \tag{35}$$

where $\vartheta \in R^q$, $\widetilde{k}(0,0,0) = 0$, $\Gamma(0,0,0) = 0$, in part of the state variables and the output ζ. In particular, static feedback $u = \widetilde{k}(x,\zeta)$ in part of the state variables and the output ζ for which the equilibrium $x = 0$, $z = 0$, $\vartheta = 0$ of the closed-loop system (34) is asymptotically stable with the attraction domain $\Omega \subset X \times Z \times R^q$, the functions $\Gamma(\cdot,\cdot,\cdot)$ and $\widetilde{k}(\cdot,\cdot,\cdot)$ being locally Lipschitzian and bounded.

The asymptotic observer for estimation of the vector x of part of the state variables of system (34) is constructed as follows:

$$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + B\phi_0(\widehat{x}, z, u), \tag{36}$$

where $\phi_0(x, z, u)$ is some locally Lipschitzian function of its arguments that is bounded in x, in particular, a function coinciding with the function $\phi(x, z, u)$ in the right-hand side of system (34), at that $\phi_0(0, 0, 0) = 0$. In observer (36), the gain matrix $L \in R^{r\times p}$ has the form $L = \mathrm{diag}\,(L^1, \ldots, L^p)$, $L^i = (\theta k_1^i, \theta^2 k_2^i, \ldots, \theta^{r_i} k_{r_i}^i)^{\mathrm{T}}$, $i = \overline{1,p}$, where $\theta > 0$ is a parameter, k_j^i, $j = \overline{1, r_i}$, $i = \overline{1,p}$, are the components of the vectors $K^i = (k_1^i, k_2^i, \ldots, k_{r_i}^i)^{\mathrm{T}}$, $i = \overline{1,p}$, such that the matrices $A^i + K^i C^i$, $i = \overline{1,p}$, have eigenvalues only with negative real parts. The following result makes use of the notation $\chi = (x^{\mathrm{T}}, z^{\mathrm{T}}, \vartheta^{\mathrm{T}})^{\mathrm{T}}$.

Theorem 4.2 ([62, 63, 139]). *Let us consider a system composed of the equations of* (34), *observer* (36), *and the system* $\dot{\vartheta} = \Gamma(\vartheta, \widehat{x}, \zeta)$ *under control* $u = \widetilde{k}(\vartheta, \widehat{x}, \zeta)$. *Let* $S \subset \Omega$ *and* $Q \subset R^r$ *be arbitrary bounded closed sets. Then,* (i) *there exists a value* $\theta_1^* > 1$ *of the parameter* θ *such that for all* $\theta \geq \theta_1^*$ *and all* $\chi(0) \in S$ *and* $\widehat{x}(0) \in Q$, *the corresponding solutions* $\chi(t)$, $\widehat{x}(t)$ *of the system are definite for all* $t \geq 0$ *and bounded;* (ii) *for any given* $\mu > 0$, *there exist* $T > 0$ *and a value* $\theta_2^* > 1$ *of the parameter* θ *depending on* μ *such that for all* $\theta \geq \theta_2^*$ *for any* $\chi(0) \in S$ *and* $\widehat{x}(0) \in Q$ *the corresponding solutions* $\chi(t)$, $\widehat{x}(t)$ *of the system satisfy the estimates* $|\chi(t)| \leq \mu$ *and* $|\widehat{x}(t)| \leq \mu$ *for all* $t \geq T$; (iii) *for any given* $\mu > 0$, *there exists a value* $\theta_3^* > 1$ *of the parameter* θ *depending on* μ *such that for all* $\theta \geq \theta_3^*$ *for any* $\chi(0) \in S$ *and* $\widehat{x}(0) \in Q$ *the corresponding solutions* $\chi(t)$, $\widehat{x}(t)$ *of the system are such that the inequality* $|\chi(t) - \chi_r(t)| \leq \mu$, *where* $\chi_r(t) = (x_r^{\mathrm{T}}(t), z_r^{\mathrm{T}}(t), \vartheta_r^{\mathrm{T}}(t))^{\mathrm{T}}$ *is the solution of system* (34) *with the control* (35) *meeting the initial condition* $\chi_r(0) = \chi(0)$, *is satisfied for all* $t \geq 0$; (iv) *if the equilibrium* $\chi = 0$ *of system* (34) *with the control* (35) *is exponentially stable, then there exists a value* $\theta_4^* > 1$ *of the parameter* θ *such that for all* $\theta \geq \theta_4^*$ *the equilibrium* $\chi = 0$, $\widehat{x} = 0$ *of the system under consideration is exponentially stable with the attraction domain containing the set* $S \times Q$.

4.3. Global stabilization of the nonlinear dynamic systems like (1) using the separation principle was considered, for example, in [5, 6, 14, 26, 64, 89, 90, 99, 104, 106, 108, 130]. The following theorem can be formulated on the basis of the results of [14, 139, 142, 172, 173].

Theorem 4.3. *Let us assume that* (i) *system* (1) *with the continuous right-hand side is detectable and there exists a continuous state feedback* $u = k(x)$, $k(0) = 0$, *which globally asymptotically stabilizes the equilibrium* $x = 0$, $u = 0$ *of system* (1); (ii) *system* (1) *with the control* $u = k(x + v)$, *where* v *is a new system input, has stable map of the input-state relative to the input* v. *Then, for the control* $u = k(\widehat{x}) = k(x + e)$, *system* (30) *with the right-hand side in point is global and asymptotical at the point* $x = 0$, $\widehat{x} = 0$.

For the affine and general cases, the problem of global stabilization of the nonlinear dynamic systems like (1) was considered, respectively, in [99] and [106, 130]. We rely on these results to formulate the following additional assumptions and theorem.

Assumption 4.1. *There exists a continuous state feedback* $u = k(x)$, $k(0) = 0$ *such that for control* $u = k(x)$ *system* (1) *is globally asymptotically stable at the point* $x = 0$ *with the Lyapunov function* $V(x)$ *satisfying for all* $x \in R^n$ *the inequalities*

$$\alpha_1(|\, x\,|) \leq V(x) \leq \alpha_2(|\, x\,|), \tag{37}$$

$$\frac{\partial V(x)}{\partial x} f(x, k(x)) \leq -\alpha_3(|\, x\,|), \tag{38}$$

where $\alpha_i(\cdot)$, $i = 1, 2$, are some functions of the class K_∞ and $\alpha_3(\cdot)$ is a function of the class K that is determined on R^+.

Assumption 4.2. *System (1) admits construction of an asymptotic observer*

$$\dot{\widehat{x}} = g(\widehat{x}, h(x), u), \quad g(0, 0, 0) = 0 \tag{39}$$

such that the equation of the error $e = \widehat{x} - x$ of system state estimation by the observer has the form (31), where the map $F : R^n \times R^n \times R^m \to R^n$ is continuous. For any fixed control u from the permissible class of control laws for which all solutions of system (1) with the given control are definite for all $t \geq 0$, the equilibrium $e = 0$ of system (31), where $x(t)$ is an arbitrary solution of system (1) with the given control, is globally asymptotically stable. For any fixed control u from the permissible class of control laws for which some solution $x(t)$ of system (1) with the given control is definite only over a finite half-interval $t \in [0, T)$, $0 < T < +\infty$, the solutions $e(t)$ of system (31) under the given u and $x(t)$ for any initial values $e(0)$ also are definite and bounded over the given time half-interval.

Theorem 4.4. *Let Assumptions 4.1 and 4.2 be satisfied for system (1) with continuous right-hand side and the inequalities*

$$\frac{\partial V(\widehat{x})}{\partial \widehat{x}}(f(\widehat{x} - e, k(\widehat{x})) - f(\widehat{x}, k(\widehat{x}))) \leq \varrho_1(|\, \widehat{x} \,|) + \varrho_2(|\, e \,|), \tag{40}$$

$$\frac{\partial V(\widehat{x})}{\partial \widehat{x}} F(\widehat{x} - e, e, k(\widehat{x})) \leq \phi_1(|\, \widehat{x} \,|) + \phi_2(|\, e \,|), \tag{41}$$

where $\varrho_i(\cdot)$ and $\phi_i(\cdot)$, $i = 1, 2$, are some functions from the class K_∞, be valid for all $\widehat{x} \in R^n$, $e \in R^n$. Then, system (30) composed of the equations of system (1) and those of observer (39) for the control $u = k(\widehat{x})$ is globally asymptotically stable at the point $x = 0$, $\widehat{x} = 0$, if $\widetilde{\alpha}(s) = \alpha_3(s) - \varrho_1(s) - \phi_1(s) > 0$ for all $s \in R^+/\{0\}$.

Theorem 4.4 is proved in the Appendix, its corollary being represented by Theorem 4.5.

Theorem 4.5 ([106, 130]). *Let* (i) *the map $f : R^n \times R^m \to R^n$ in the right-hand side of system (1) be continuously differentiable and global Lipschitzian in x uniformly in u with a constant γ_f;* (ii) *Assumption 4.2 be satisfied and the map $F : R^n \times R^n \times R^m \to R^n$ be global Lipschitzian in e uniformly in u and x with a constant γ_F; and* (iii) *there be a continuously differentiable state feedback $u = k(x)$, $k(0) = 0$ which globally exponentially stabilizes the equilibrium $x = 0, u = 0$ of system (1). Then, for the control $u = k(\widehat{x})$ system (30) consisting of the equations of system (1) and the equations of observer (39) is globally asymptotically stable at the point $x = 0$, $\widehat{x} = 0$.*

Theorem 4.5 is proved in the Appendix. We note that under additional assumptions its conditions (i), (ii), and (iii) are satisfied for the classes of dynamic systems like (1) that admit construction of the global asymptotic observers by means of the aforementioned basic methods of observer construction for the nonlinear dynamic systems with control.

Theorems 4.3, 4.4 and 4.5 imply that the separation principle is valid in the case of global asymptotic stabilization of some classes of nonlinear dynamic systems and, for the corresponding classes, answer positively to the questions formulated in the Introduction about global asymptotic stabilization of the system by means of separate construction of the stabilizing state feedback and the observer for estimation of the system state with subsequent substitution of the state estimate into the feedback.

Note 4.1. By virtue of $e = \widehat{x} - x$, asymptotic stability of the equilibrium $x = 0$, $\widehat{x} = 0$ of system (30) under an arbitrary fixed control $u = k(\widehat{x})$ from the permissible class of control laws is equivalent to stability of the equilibrium $x = 0$, $e = 0$ of the system

$$\dot{x} = f(x, k(x + e)), \quad \dot{e} = F(x, e, k(x + e))$$

consisting of the equations of system (1) with the given control and the equations of system (31) describing dynamics of the state estimate error $e = \widehat{x} - x$ of (1) under the given control, as well as to asymptotic stability of the equilibrium $\widehat{x} = 0$, $e = 0$ of the system

$$\dot{\widehat{x}} = g(\widehat{x}, h(\widehat{x} - e), k(\widehat{x})), \quad \dot{e} = F(\widehat{x} - e, e, k(\widehat{x})) \tag{42}$$

consisting of the equations of observer (39) with the given control and the equations of system (31) under the given control. We note that, by virtue of $e = \widehat{x} - x$, any solution $x(t)$ of system (1) with the control $u = k(\widehat{x})$ is representable as $x(t) = \widehat{x}(t) - e(t)$, where $\widehat{x}(t)$, $e(t)$ is the corresponding solution of system (42). Let the equilibrium $\widehat{x} = 0$, $e = 0$ of system (42) be globally asymptotically stable. Then, the inequality

$$|x(t)| = |x(t) - \widehat{x}(t) + \widehat{x}(t)| \leq |\widehat{x}(t)| + |\widehat{x}(t) - x(t)| = |\widehat{x}(t)| + |e(t)|$$

means that for an arbitrary solution $x(t)$ of system (1) with the control $u = k(\widehat{x})$ $x(t) \to 0$ is satisfied for $t \to +\infty$.

4.4. For system stabilization by the dynamic output feedbacks, the so-called method of backstepping in observer [9, 137] offers an alternative to the separation principle. This method also makes use of the system state estimate made by the observer and relies on direct stabilization of a special extended system consisting of the equations of the original system and the equations of the error of system state estimate made by the observer and uses the control laws in the form of the state functions of the observer and the output of the original system.

Let us consider the case of a smooth nondegenerate change of variables like $x = T(\chi)$, $\chi = T^{-1}(x)$, $T^{-1}(0) = 0$, where the corresponding functions are defined globally, which rearranges the dynamic system (1) with the scalar input and output in

$$\dot{\chi} = A\chi + \varphi(\chi_1) + \widetilde{B}(\chi_1)u, \quad y = \chi_1, \tag{43}$$

where $\chi = (\chi_1, \ldots, \chi_n)^{\mathrm{T}} \in R^n$ is the system state vector; $u \in R$ is the control; $y \in R$ is the measured system output; $A = (a_{ij})$, $i = \overline{1,n}$, $j = \overline{1,n}$, is a square n-order matrix with the elements $a_{ij} = 1$, if $j - i = 1$, and $a_{ij} = 0$, if $j - i \neq 1$; $\varphi(\chi_1) = (\varphi_1(\chi_1), \ldots, \varphi_n(\chi_1))^{\mathrm{T}}$, $\varphi_i(\cdot) \in C^\infty(R)$, $\varphi_i(0) = 0$, $i = \overline{1,n}$; the vector function $\widetilde{B}(\chi_1) = (0, \ldots, 0, \beta(\chi_1))^{\mathrm{T}}$ is an n-dimensional vector, the function $\beta : R \to R$ being such that $\beta(y) \neq 0$ for any $y \in R$.

System (43) may be regarded as a special case of system (9). For system (43), the global asymptotic observer has a form similar to (10):

$$\dot{\widehat{\chi}} = A\widehat{\chi} + LC(\widehat{\chi} - \chi) + \varphi(\chi_1) + \widetilde{B}(\chi_1)u, \tag{44}$$

where $C = (1, 0, \ldots, 0)$ is the row vector of length n. The vector $L = (l_1, \ldots, l_n)^{\mathrm{T}} \in R^n$ in observer (44) defines dynamics of the error $e = \widehat{\chi} - \chi$ of state estimate of system (43) and is taken so that the eigenvalues of $A + LC$ have only negative real parts. Such a choice of L is feasible because the pair (A, C) is observable. For systems (43) and (44), control is identical, belongs to the permissible class of control laws, and is such that the solutions $\chi(t)$ of system (43) with the given control are defined for all $t \geq 0$ for any $\chi(0)$.

Under the same control in systems (43) and (44), the equation of error $e = \widehat{\chi} - \chi$ of estimation by observer (44) of the state of system (43) has the form (11). Consequently, by virtue of the choice of the spectrum of the matrix $A + LC$, the equilibrium $e = 0$ of system (11) with the right-hand side at issue is globally exponentially stable with the Lyapunov function like $W(e) = e^{\mathrm{T}}Pe$, where, by virtue of the aforementioned choice of the spectrum of the matrix $A + LC$, the matrix $P = P^{\mathrm{T}} > 0$ satisfies the Lyapunov equation $(A + LC)^{\mathrm{T}} P + P(A + LC) = -Q$, $Q = Q^{\mathrm{T}} > 0$ whose solution exists. For the time derivative of the function $W(e)$, by virtue of system (11) with the right-hand side at issue for all $e \in R^n$, the following estimate is valid:

$$\dot{W}(e)\Big|_{(11)} = -e^{\mathrm{T}} Q e \le -\lambda_{\min}(Q)|\, e\,|^2 \stackrel{def}{=} -\lambda|\, e\,|^2.$$

Let us consider the system

$$\dot{\chi} = A\chi + \varphi(\chi_1) + \widetilde{B}(\chi_1)u, \quad \dot{e} = (A + LC)e, \quad y = \chi_1, \tag{45}$$

composed of the equations of system (43) and the equations of the system describing dynamics of the error $e = \widehat{\chi} - \chi$ of estimation by observer (44) of the state of system (43). The relation $\chi = \widehat{\chi} - e$ enables one to represent system (45) as follows:

$$\begin{gathered}
\dot{\chi}_1 = \widehat{\chi}_2 - e_2 + \varphi_1(\chi_1),\\
\dot{\widehat{\chi}}_2 = \widehat{\chi}_3 + l_2 e_1 + \varphi_2(\chi_1),\\
\vdots\\
\dot{\widehat{\chi}}_{n-1} = \widehat{\chi}_n + l_{n-1}e_1 + \varphi_{n-1}(\chi_1),\\
\dot{\widehat{\chi}}_n = l_n e_1 + \varphi_n(\chi_1) + \beta(\chi_1)u,\\
\dot{e} = (A + LC)e,\\
y = \chi_1,
\end{gathered} \tag{46}$$

where $\widehat{\chi} = (\widehat{\chi}_1, \ldots, \widehat{\chi}_n)^{\mathrm{T}} \in R^n$ is the observer state vector (44), $e_1 = \widehat{\chi}_1 - \chi_1$, $e_2 = \widehat{\chi}_2 - \chi_2$.

The control law $u = \widetilde{k}(\widehat{\chi}, y)$, $\widetilde{k}(0, 0) = 0$, which is determined by the backstepping in observer and globally asymptotically stabilizes the equilibrium $\widehat{\chi} = 0$, $e = 0$, $u = 0$ of system (46) and, consequently, by virtue of $\chi = \widehat{\chi} - e$, also the equilibrium $\chi = 0$, $e = 0$, $u = 0$ of system (45), is as follows [9, 137]:

$$\begin{gathered}
u = \alpha_n(\widehat{\chi}, \chi_1) = \frac{1}{\beta(\chi_1)}\Bigg(-c_n z_n - z_{n-1} - l_n(\widehat{\chi}_1 - \chi_1) - \varphi_n(\chi_1)\\
+ \frac{\partial\alpha_{n-1}}{\partial\chi_1}\Big(\widehat{\chi}_2 + \varphi_1(\chi_1)\Big) + \sum_{i=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\widehat{\chi}_i}\dot{\widehat{\chi}}_i - d_n\left(\frac{\partial\alpha_{n-1}}{\partial\chi_1}\right)^2 z_n\Bigg),\\
\alpha_k(\widehat{\chi}_1, \ldots, \widehat{\chi}_k, \chi_1) = -c_k z_k - z_{k-1} - l_k(\widehat{\chi}_1 - \chi_1) - \varphi_k(\chi_1)\\
+ \frac{\partial\alpha_{k-1}}{\partial\chi_1}\Big(\widehat{\chi}_2 + \varphi_1(\chi_1)\Big) + \sum_{i=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial\widehat{\chi}_i}\dot{\widehat{\chi}}_i - d_k\left(\frac{\partial\alpha_{k-1}}{\partial\chi_1}\right)^2 z_k,\ k = \overline{3, n-1},\\
\alpha_2(\widehat{\chi}_1, \widehat{\chi}_2, \chi_1) = -c_2 z_2 - z_1 - l_2(\widehat{\chi}_1 - \chi_1) - \varphi_2(\chi_1) + \frac{\partial\alpha_1}{\partial\chi_1}\Big(\widehat{\chi}_2 + \varphi_2(\chi_1)\Big) - d_2\left(\frac{\partial\alpha_1}{\partial\chi_1}\right)^2 z_2,\\
\alpha_1(\chi_1) = -(c_1 + d_1)z_1 - \varphi_1(\chi_1),\\
z_1 = \chi_1, \quad z_2 = \widehat{\chi}_2 - \alpha_1(\chi_1), \quad z_3 = \widehat{\chi}_3 - \alpha_2(\widehat{\chi}_1, \widehat{\chi}_2, \chi_1),\ \ldots,\\
z_n = \widehat{\chi}_n - \alpha_{n-1}(\widehat{\chi}_1, \ldots, \widehat{\chi}_{n-1}, \chi_1).
\end{gathered} \tag{47}$$

Here, $c_i > 0$, $i = \overline{1, n}$, are arbitrary positive constants, and the constants $d_i > 0$, $i = \overline{1, n}$, are chosen so that the inequality

$$\lambda > \max\left\{\frac{1}{4d_1}, \frac{1}{4d_2}, \ldots, \frac{1}{4d_n}\right\} \tag{48}$$

is satisfied. We note that condition (48) may be satisfied, for example, by fixing the coefficients $d_i > 0$, $i = \overline{1, n}$, and selecting the matrix Q in the Lyapunov equation so as to satisfy this condition. According to Note 4.1, it follows from the global asymptotic stability of the equilibrium $\chi = 0$, $e = 0$ of system (45) with control (47) that $\chi(t) \to 0$ is satisfied for $t \to \infty$ for an arbitrary solution $\chi(t)$ of system (43) under control (47). An algorithm to construct control (47) by the method of backstepping in observer is presented in the Appendix.

We note that construction of the stabilizing control laws as the dynamic output feedbacks by means of the method of backstepping in observer for the systems with vector input and output was considered, for example, in [58, 137]. This method was extended to the global stabilization of the nonlinear systems with linear zero-dynamics system in the cases of minimum phase and non-minimum phase, respectively, in [9, 137] and [66,131].

Besides the method of backstepping in observer, the method based on system linearization by the dynamic output feedback [7] offers a possible approach to global stabilization of systems like (43) by the dynamic output feedbacks. This method is based on using change of variables and the dynamic output feedback to rearrange system like (43) in a linear observable stabilizable system and then stabilize the resulting linear system by means of the feedback in the state estimate made by the observer. Other methods of stabilization of the nonlinear dynamic systems of various lower triangular forms by the dynamic output feedbacks were discussed, for example, in [10, 19, 22, 37, 38, 41, 82, 102, 119, 124].

5. CONCLUSIONS

The present review considered the main methods of constructing the global asymptotic observers for the nonlinear dynamic systems with control, as well as the main approaches to the asymptotic stabilization of the equilibria of the nonlinear dynamic system by means of the dynamic output feedbacks using the estimate of system state made by the asymptotic observer, namely, utilization of the separation principle and the method of backstepping in observer.

Presented were the classes of the nonlinear dynamic systems admitting construction of the global asymptotic observers, as well as the classes of systems for which the separation principle is valid, that is, asymptotic stabilization of the given system equilibrium is possible by separate construction of the stabilizing state feedback and the observer to estimate the system state with subsequent substitution of the state estimate in the feedback.

We note that in the general case the problem of constructing the global asymptotic observer of the nonlinear dynamic system remains still undecided and the global observers were established only for some classes of nonlinear systems. The possibility of using the separation principle in the case of global asymptotic system stabilization has been demonstrated only for some classes of the nonlinear systems. In the general case, the separation principle does not work for the global asymptotic stabilization of nonlinear dynamic systems. Examples are known of nonlinear systems to which in the case of global stabilization the separation principle is inapplicable despite the global stabilizability of the system by the state feedback and the possibility of estimating the system state by the global asymptotic observer.

The paper also described an algorithm of global asymptotic stabilization of the equilibria of the nonlinear dynamic systems using the backstepping method in an observer based on direct stabilization of a special extended system consisting of the equations of the original system and the equations of the error of system state estimation by the observer by means of the control laws having the form of the functions of observer state and the output of the original system. We note that this algorithm is applicable to the problems of global asymptotic stabilization of a fairly narrow class of special nonlinear systems.

The problems of stabilization were considered for no uncertainties in the system. However, some of the above approaches work in the case of, for example, parametric system uncertainties. The method of backstepping in observer was generalized in [137] to the case where the right-hand side of the system has unknown parameters. The separation principle can be used for asymptotic stabilization of systems with unknown parameters only for a very narrow class of nonlinear systems admitting construction of the adaptive observers (see, for example, [174]).

For control of wider classes of nonlinear systems both with and without uncertainties, topical are, therefore, further extension of the classes of nonlinear dynamic systems to which the separation principle can be applied in the case of global asymptotic system stabilization, as well as extension of the existing and development of new methods of constructing global asymptotic observers for the nonlinear dynamic systems with control and methods of system stabilization by the dynamic output feedbacks.

We note that the major part of the aforementioned results concerning asymptotic stabilization of the system equilibria using the system state estimate made by the asymptotic observer are applicable to the problems of tracking the program system trajectories.

APPENDIX

Proof of Theorem 3.1. The positive definite function $W(e) = e^{\mathrm{T}}Pe$, where the matrix $P = P^{\mathrm{T}} > 0$ is the solution of the Lyapunov equation (17) for some fixed matrix $Q = Q^{\mathrm{T}} > 0$, is considered as the Lyapunov function of system (16) describing the dynamics of error $e = \widehat{x} - x$ of the state estimate of system (14) by observer (15) for an arbitrary fixed control u meeting the theorem's conditions. According to them and the properties of the quadratic forms, the function $W(e)$ satisfies the following inequalities for any $e \in R^n$:

$$\lambda_{\min}(P)|\, e\,|^2 \le W(e) \le \lambda_{\max}(P)|\, e\,|^2, \quad \left|\frac{\partial W(e)}{\partial e}\right| = |2e^{\mathrm{T}}P| \le 2\lambda_{\max}(P)|\, e\,|, \tag{A.1}$$
$$\frac{\partial W(e)}{\partial e}(A + LC)e = -e^{\mathrm{T}}Qe \le -\lambda_{\min}(Q)|\, e\,|^2.$$

Then, the time derivative of the function $W(e)$ along the trajectories of system (16) is representable for any $e \in R^n$ and for all $t \ge 0$ as

$$\dot{W}(e)\Big|_{(16)} = 2e^{\mathrm{T}}P\dot{e} = -e^{\mathrm{T}}Qe + 2e^{\mathrm{T}}P\left[\widetilde{f}(x(t)+e,u) - \widetilde{f}(x(t),u)\right]$$
$$\le -\lambda_{\min}(Q)|\, e\,|^2 + 2\gamma_{\widetilde{f}}\lambda_{\max}(P)|\, e\,|^2 = (-\lambda_{\min}(Q) + 2\gamma_{\widetilde{f}}\lambda_{\max}(P))|\, e\,|^2.$$

For $\gamma_{\widetilde{f}} < \lambda_{\min}(Q)/(2\lambda_{\max}(P))$, the following inequality is true:

$$\dot{W}(e)\Big|_{(16)} < 0, \quad e \ne 0.$$

Consequently, according to [175], the equilibrium $e = 0$ of system (16) for the given control u and arbitrary solution $x(t)$ of system (14) with the given control is globally exponentially stable for $\gamma_{\widetilde{f}} < \lambda_{\min}(Q)/(2\lambda_{\max}(P))$, and for any initial values $e(0)$ the solutions $e(t)$ of system (16) are definite for all $t \ge 0$ and for all $t \ge 0$ satisfy the inequality

$$|e(t)| \le \beta |e(0)| e^{-\alpha t},$$

where $\beta = \sqrt{\dfrac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ and $\alpha = \dfrac{\lambda_{\min}(Q) - 2\gamma_{\widetilde{f}}\lambda_{\max}(P)}{2\lambda_{\max}(P)}$.

Proof of Theorem 3.2. Let us consider the linear change of variables $z_i = \theta^{1-i} e_i$, $i = \overline{1, n}$. In terms of the new variables z_i, $i = \overline{1, n}$, and with regard for $l_i = \theta^i k_i$, system (20) describing dynamics of the error $e = \widehat{x} - x$ of state estimation of system (18) by observer (19) takes on the form

$$\dot{z} = \theta A_C z + \widetilde{G}(z, x(t), u), \tag{A.2}$$

where

$$z = (z_1, \ldots, z_n)^{\mathrm{T}},$$

$$\widetilde{G}(z, x(t), u) = \left(g_1(z_1, x(t), u), \frac{1}{\theta} g_2(z_1, \theta z_2, x(t), u), \ldots, \frac{1}{\theta^{n-1}} g_n(z_1, \theta z_2, \ldots, \theta^{n-1} z_n, x(t), u)\right)^{\mathrm{T}}.$$

We consider the quadratic function $W(z) = \frac{1}{\theta} z^{\mathrm{T}} P_C z$ as the Lyapunov function of system (A.2) under an arbitrary fixed control u satisfying the theorem's conditions and an arbitrary solution $x(t)$ of system (18) with the given control. Here, the matrix $P_C = P_C^{\mathrm{T}} > 0$ satisfies the Lyapunov equation $A_C^{\mathrm{T}} P_C + P_C A_C = -Q_C$ for some fixed matrix $Q_C = Q_C^{\mathrm{T}} > 0$ whose solution exists because, according to the conditions of the theorem, the matrix A_C has eigenvalues only with negative real parts. We note that for any $z \in R^n$ the function $W(z)$ satisfies estimates similar to (A.1) that are valid for the positive definite quadratic forms.

For any $z \in R^n$ and all $t \geq 0$, the time derivative of the function $W(z)$ along the trajectories of system (A.2) is given by

$$\begin{aligned} \dot{W}(z)\Big|_{(\mathrm{A.2})} &= \frac{1}{\theta} 2 z^{\mathrm{T}} P_C \dot{z} = -z^{\mathrm{T}} Q_C z + \frac{1}{\theta} 2 z^{\mathrm{T}} P_C \widetilde{G}(z, x(t), u) \\ &\leq -\lambda_{\min}(Q_C) |\, z\,|^2 + \frac{1}{\theta} 2 \lambda_{\max}(P_C) |\, z\,| |\widetilde{G}(z, x(t), u)|. \end{aligned}$$

We note that, as follows from the global Lipschitzness in x of the function $a_n(x)$ and the uniform in u global Lipschitzness in x of the functions $b_i(x, u)$, $i = \overline{1, n}$, in the right-hand side of system (18), the functions $g_i(e, x, u)$, $i = \overline{1, n}$, in the right-hand side of system (20) are globally Lipschitzian in e uniformly in x and u. Then, with regard for the fact that the equalities $g_i(0, 0, \ldots, 0, x, u) = 0$, $i = \overline{1, n}$, are valid for all $x \in R^n$, $u \in R^m$, the following estimates are valid for the functions $g_i(z_1, \theta z_2, \ldots, \theta^{i-1} z_i, x, u)$, $i = \overline{1, n}$, $\theta > 1$, in the right-hand side of system (A.2):

$$\begin{aligned} &\left|\frac{1}{\theta^{i-1}} g_i(z_1, \theta z_2, \ldots, \theta^{i-1} z_i, x, u)\right| \\ &\leq \frac{1}{\theta^{i-1}} \gamma_{g_i} \sqrt{z_1^2 + \theta^2 z_2^2 + \ldots + \theta^{2(i-1)} z_i^2} \leq \gamma_{g_i} |\, z\,|, \quad i = \overline{1, n}, \end{aligned} \tag{A.3}$$

where γ_{g_i}, $i = \overline{1, n}$, are the corresponding Lipschitz constants.

We make use of inequalities (A.3) to obtain the following estimate of the time derivative of $W(z)$ along the trajectories of system (A.2):

$$\dot{W}(z)\Big|_{(\mathrm{A.2})} \leq \left(-\lambda_{\min}(Q_C) + \frac{\gamma_G}{\theta} 2 \lambda_{\max}(P_C)\right) |\, z\,|^2,$$

where $\gamma_G = n \max \gamma_{g_i}$. For $\theta > \theta_* = \max\{1.2 \gamma_G \lambda_{\max}(P_C)/\lambda_{\min}(Q_C)\}$, the following inequality is true:

$$\dot{W}(z)\Big|_{(\mathrm{A.2})} < 0, \quad z \neq 0.$$

For the aforementioned choice of the parameter θ, the equilibrium $z = 0$ of system (A.2), according to [175], is globally exponentially stable under the given control u and an arbitrary solution $x(t)$ of system (18) with the given control. Now, the global exponential stability of the equilibrium $e = 0$ of system (20) with the quadratic Lyapunov function $W(e) = e^{\mathrm{T}}Pe$, where $P = (p_{ij})$, $i = \overline{1,n}$, $j = \overline{1,n}$, is the quadratic n-order matrix with the elements $p_{ij} = p_{C_{ij}}/\theta^{i+j-1}$, $i = \overline{1,n}$, $j = \overline{1,n}$, follows from the linearity and form of the change of variables $z_i = \theta^{1-i}e_i$, $i = \overline{1,n}$. Consequently, there exist constants $\alpha > 0$ and $\beta > 0$ that are independent of u and $x(t)$ and are such that for any initial values $e(0)$ the solutions $e(t)$ of system (20) under the given control u and an arbitrary solution $x(t)$ of system (18) with the given control for the aforementioned choice of the parameter θ for all $t \geq 0$ are definite and for all $t \geq 0$ satisfy the inequality $|e(t)| \leq \beta |e(0)| \exp(-\alpha t)$.

Proof of Theorem 3.4. Let us consider the positive definite function $W(e) = e^{\mathrm{T}}Pe$, where the matrix $P = P^{\mathrm{T}} > 0$ that was taken from the theorem's conditions satisfies the matrix inequality (27), as the Lyapunov function for system (26) describing dynamics of the error $e = \widehat{x} - x$ of state estimate of system (22) by observer (24) for an arbitrary fixed control u meeting the theorem's conditions. The time derivative of the function $W(e)$ along the trajectories of system (26) is representable for any $e \in R^n$ and all $t \geq 0$ as follows:

$$\dot{W}(e)\Big|_{(26)} = \begin{bmatrix} e \\ \Delta(t)\eta \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (A+LC)^{\mathrm{T}}P + P(A+LC) & PG \\ G^{\mathrm{T}}P & 0 \end{bmatrix} \begin{bmatrix} e \\ \Delta(t)\eta \end{bmatrix}$$
$$\leq \begin{bmatrix} e \\ \Delta(t)\eta \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\nu I & -(H+KC)^{\mathrm{T}} \\ -(H+KC) & -D \end{bmatrix} \begin{bmatrix} e \\ \Delta(t)\eta \end{bmatrix}.$$

Taking into account $(H + KC)e = \eta$, we obtain

$$\dot{W}(e)\Big|_{(26)} \leq -\nu |\, e\,|^2 - 2\eta^{\mathrm{T}}\Delta(t)\Big(I - \frac{1}{b}\Delta(t)\Big)\eta.$$

Since $\delta_i(t) \in [0, b]$, $i = \overline{1,r}$, the inequality

$$\dot{W}(e)\Big|_{(26)} \leq -\nu |\, e\,|^2$$

is valid. We note that for any $e \in R^n$ the function $W(e)$ also satisfies the estimates (A.1). Consequently, according to [175], the equilibrium $e = 0$ of system (26) is globally exponentially stable, and for any initial conditions $e(0)$ the solutions $e(t)$ of system (16) are definite for all $t \geq 0$ and for all $t \geq 0$ satisfy the inequality

$$|e(t)| \leq \beta |e(0)| e^{-\alpha t},$$

where $\beta = \sqrt{\dfrac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ and $\alpha = \dfrac{\nu}{2\lambda_{\max}(P)}$.

Proof of Theorem 4.4. Let us consider system (30) under the control $u = k(\widehat{x})$. By the linear change of variables

$$\widehat{x} = \widehat{x}, \quad e = \widehat{x} - x, \tag{A.4}$$

under the given control system (30) with the right-hand side in point is rearranged to a form which, with regard for $g(\widehat{x}, h(x), k(\widehat{x})) = f(x, k(\widehat{x})) + F(x, e, k(\widehat{x}))$, is representable as follows:

$$\dot{\widehat{x}} = f(\widehat{x} - e, k(\widehat{x})) + F(\widehat{x} - e, e, k(\widehat{x})), \quad \dot{e} = F(\widehat{x} - e, e, k(\widehat{x})). \tag{A.5}$$

The subsystem of equations in $\widehat{x}$ of system (A.5) is given by

$$\dot{\widehat{x}} = f(\widehat{x} - e, k(\widehat{x})) + F(\widehat{x} - e, e, k(\widehat{x})) = \Phi(\widehat{x}, e). \tag{A.6}$$

The function $V_1(\widehat{x}) = V(\widehat{x})$, where $V(x)$ is the Lyapunov function of system (1) that is closed by the control $u = k(x)$, is considered as the Lyapunov function of system (A.6) with the perturbation e. By replacing x in (38) by $\widehat{x}$ and taking into account the resulting inequality and inequalities (40) and (41), the time derivative of V_1 along the trajectories of system (A.6) for any $\widehat{x} \in R^n$ and $e \in R^n$ can be given by

$$\dot{V}_1(\widehat{x})\Big|_{(\mathrm{A.6})} = \frac{\partial V_1(\widehat{x})}{\partial \widehat{x}} f(\widehat{x}, k(\widehat{x})) + \frac{\partial V_1(\widehat{x})}{\partial \widehat{x}} \Big(f(\widehat{x} - e, k(\widehat{x})) - f(\widehat{x}, k(\widehat{x})) \Big)$$
$$+ \frac{\partial V_1(\widehat{x})}{\partial \widehat{x}} F(\widehat{x} - e, e, k(\widehat{x})) \le -\alpha_3(|\,\widehat{x}\,|) + \varrho_1(|\,\widehat{x}\,|) + \varrho_2(|\,e\,|) + \phi_1(|\,\widehat{x}\,|) + \phi_2(|\,e\,|).$$

For all $\widehat{x} \in R^n$ and $e \in R^n$ we now have

$$\begin{aligned} \dot{V}_1(\widehat{x})\Big|_{(\mathrm{A.6})} &\le -(\alpha_3(|\,\widehat{x}\,|) - \varrho_1(|\,\widehat{x}\,|) - \phi_1(|\,\widehat{x}\,|)) + (\varrho_2(|\,e\,|) + \phi_2(|\,e\,|)) \\ &= -\widetilde{\alpha}(|\,\widehat{x}\,|) + \widetilde{\varrho}(|\,e\,|), \end{aligned} \tag{A.7}$$

where $\widetilde{\alpha}(s) = \alpha_3(s) - \varrho_1(s) - \phi_1(s)$, $\widetilde{\varrho}(s) = \varrho_2(s) + \phi_2(s)$, $s \in R^+$.

We note that, according to Assumption 4.2, any solution $e(t)$ of system (31) with the control $u = k(\widehat{x}) = k(x + e)$ is definite at least over some finite time half-interval $t \in [0, T)$, $0 < T < +\infty$, and continuous and bounded over this half-interval. Inequalities (37) (with regard for the change of x by $\widehat{x}$) and (A.7) mean that for any $\widehat{x}(0)$ a bounded solution $\widehat{x}(t)$ of system (A.6) corresponds to any bounded perturbation $e(t)$ and, by virtue of $x = \widehat{x} - e$, also a bounded solution $x(t)$ of (1) with the control $u = k(x + e)$, where $u = k(x)$ is taken from the formulation of the theorem. For any initial values $x(0)$, the solutions $x(t)$ of system (1) with the control $u = k(x + e)$ are then definite for all $t \ge 0$, and any solution $e(t)$ of system (31) with the given control $u = k(x + e)$ is definite for all $t \ge 0$. By virtue of $\widehat{x} = x + e$, the solutions $\widehat{x}(t)$ of system (A.6) for any $\widehat{x}(0)$ also are definite for all $t \ge 0$.

As follows from inequalities (37) (with regard for the replacement of x by $\widehat{x}$) and (A.7) with the assumption that $\widetilde{\alpha}(s) > 0$ for all $s \in R^+/\{0\}$ (see, for example, [137, 142, 176]), system (A.6) has a stable input-state map [148, 149] relative to the input e generated by system (31) with the control $u = k(\widehat{x})$, that is, for any $\widehat{x}(0)$ and arbitrary solution $e(t)$ of system (31) with the control $u = k(\widehat{x})$ the solutions $\widehat{x}(t)$ of system (A.6) exist for all $t \ge 0$ and satisfy the inequality

$$|\widehat{x}(t)| \le \beta(|\widehat{x}(0)|, t) + \gamma\left(\sup_{0 \le \tau \le t} |e(\tau)| \right), \quad \forall t \ge 0, \tag{A.8}$$

where β is a function from the class KL and γ is a function of the class K (for $d = +\infty$). Now, since any solution $e(t)$ of system (31) with the control $u = k(\widehat{x})$ is definite for all $t \ge 0$, it follows that, according to the conditions of the theorem, the equilibrium $e = 0$ of system (31) with the given control is globally asymptotically stable. Then, it follows from inequality (A.8) (see, for example, [139, 142]) that the equilibrium $\widehat{x} = 0$, $e = 0$ system (A.5) also is globally asymptotically stable. Further, it follows from (A.4) that the equilibrium $x = 0$, $\widehat{x} = 0$, of system (30) with the right-hand side at issue is globally asymptotically stable under the control $u = k(\widehat{x})$.

Proof of Theorem 4.5. Let us consider system (A.5) which is composed of the equations of observer (39) with the control $u = k(\widehat{x})$ and the equations of system (31) describing the dynamics of error $e = \widehat{x} - x$ of estimation by observer (39) of the state of system (1) with the given control and satisfies the theorem. Since, according to the theorem's conditions, system (1), which is closed by the feedback $u = k(x)$, is globally exponentially stable at the point $x = 0$ and its right-hand side without output is continuously differentiable, there exists [175] a Lyapunov function $V(x)$ such that the inequalities

$$c_1|\, x\,|^2 \le V(x) \le c_2|\, x\,|^2,$$
$$\left|\frac{\partial V(x)}{\partial x}\right| \le c_3|\, x\,|, \quad \frac{\partial V(x)}{\partial x} f(x, k(x)) \le -c_4|\, x\,|^2, \tag{A.9}$$

where $c_1,\ c_2,\ c_3,\ c_4$ are some positive constants, are satisfied for any $x \in R^n$. By using the Young inequality

$$xy \le \frac{1}{\epsilon} x^p + \epsilon^{\frac{1}{p-1}} y^{\frac{p}{p-1}} \quad \forall x, y \in R^+, \quad \forall p > 1, \quad \forall \epsilon > 0$$

and replacing x in (A.9) by $\widehat{x}$, we obtain for all $\widehat{x} \in R^n$ and $e \in R^n$ with regard for the resulting inequalities that

$$\frac{\partial V(\widehat{x})}{\partial \widehat{x}} \Big(f(\widehat{x} - e, k(\widehat{x})) - f(\widehat{x}, k(\widehat{x})) \Big)$$
$$\le c_3\gamma_f|\, e\,||\, \widehat{x}\,| \le \frac{1}{\epsilon} c_3\gamma_f|\, \widehat{x}\,|^2 + \epsilon c_3\gamma_f|\, e\,|^2 = \varrho_1(|\, \widehat{x}\,|) + \varrho_2(|\, e\,|),$$
$$\frac{\partial V(\widehat{x})}{\partial \widehat{x}} F(\widehat{x} - e, e, k(\widehat{x})) \le c_3\gamma_F|\, e\,||\, \widehat{x}\,|$$
$$\le \frac{1}{\epsilon} c_3\gamma_F|\, \widehat{x}\,|^2 + \epsilon c_3\gamma_F|\, e\,|^2 = \phi_1(|\, \widehat{x}\,|) + \phi_2(|\, e\,|).$$

Here, $\varrho_1(s) = \frac{1}{\epsilon} c_3\gamma_f s^2$, $\varrho_2(s) = \epsilon c_3\gamma_f s^2$, $\phi_1(s) = \frac{1}{\epsilon} c_3\gamma_F s^2$, $\phi_2(s) = \epsilon c_3\gamma_F s^2$, $s \in R^+$, and $\epsilon > 0$ is an arbitrary positive constant.

Now, for $\epsilon > (c_3\gamma_f + c_3\gamma_F)/c_4$ the inequality

$$\widetilde{\alpha}(s) = \alpha_3(s) - \varrho_1(s) - \phi_1(s) = c_4 s^2 - \frac{1}{\epsilon} c_3\gamma_f s^2 - \frac{1}{\epsilon} c_3\gamma_F s^2 > 0$$

is valid for all $s \in R^+/\{0\}$. Consequently, according to Theorem 4.4, for $\epsilon > (c_3\gamma_f + c_3\gamma_F)/c_4$ the equilibrium $x = 0$, $\widehat{x} = 0$, of system (30) with the right-hand side at issue is globally asymptotically stable under the control $u = k(\widehat{x})$.

Algorithm to Construct the Stabilizing Control by Means of the Backstepping Method in Observer

Step 1. Consider the first equation of system (46)

$$\dot{\chi}_1 = \widehat{\chi}_2 - e_2 + \varphi_1(\chi_1).$$

If $\widehat{\chi}_2$ were a control, then it would be possible to compensate the uncertainty of e_2 and make the variable $\chi_1(t)$ tend to zero for $t \to +\infty$ by taking some control law $\widehat{\chi}_2 = \alpha_1(\chi_1)$ in the first function of system (46). Let us construct this "virtual" control law $\widehat{\chi}_2 = \alpha_1(\chi_1)$. To this end, we make use of the following positive definite function

$$V_1(z_1, e) = \frac{1}{2} z_1^2 + W(e) > 0 \quad \text{for} \quad z_1 \ne 0,\ e \ne 0,$$

where $z_1 = \chi_1$, $W(e)$ is the Lyapunov function of system (11) with the right-hand side under consideration. Below we also use for convenience the variable $z_2 = \widehat{\chi}_2 - \alpha_1(\chi_1)$.

The time derivative of the function V_1 along the trajectories of system (46) is as follows:

$$\begin{gathered}\dot{V}_1|_{(46)} = z_1\dot{z}_1|_{(46)} + \dot{W}(e)|_{(46)} \\ = z_1\dot{z}_1|_{(46)} + \dot{W}(e)|_{(11)} \leq z_1(\widehat{\chi}_2 + \phi_1(\chi_1) - e_2) - \lambda|\, e\,|^2 \\ \leq z_1(z_2 + \alpha_1(\chi_1) + \varphi_1(\chi_1) - e_2) - \lambda e_2^2.\end{gathered}$$

By taking $\alpha_1(\chi_1) = -(c_1+d_1)z_1 - \varphi_1(\chi_1)$, where $c_1 > 0$ and $d_1 > 0$ are arbitrary positive constants, we obtain

$$\begin{gathered}\dot{V}_1|_{(46)} \leq z_1(-c_1z_1 + z_2 - d_1z_1 - e_2) - \lambda e_2^2 \\ = -c_1z_1^2 + z_1z_2 - (d_1z_1^2 + z_1e_2 + \lambda e_2^2) = -c_1z_1^2 + z_1z_2 - S_1,\end{gathered}$$

where

$$S_1 = \left(\sqrt{d_1}z_1 + \frac{1}{2\sqrt{d_1}}e_2\right)^2 + \left(\lambda - \frac{1}{4d_1}\right)e_2^2 \geq 0 \quad \text{for} \quad \lambda > \frac{1}{4d_1}.$$

Consequently, for an appropriate choice of the constant d_1, the upper estimate of the time derivative of the function V_1 along the trajectories of system (46) consists of the negative definite part and the alternating-sign term z_1z_2.

Step 2. Consider a subsystem consisting the two first equations of system (46) and representable as

$$\begin{gathered}\dot{\chi}_1 = \alpha_1(\chi_1) + \varphi_1(\chi_1) - e_2 + (\widehat{\chi}_2 - \alpha_1(\chi_1)), \\ \dot{\widehat{\chi}}_2 = \widehat{\chi}_3 + l_2(\widehat{\chi}_1 - \chi_1) + \varphi_2(\chi_1).\end{gathered}$$

At this step, we assume that it is already $\widehat{\chi}_2$ rather than $\widehat{\chi}_3$ that is the "virtual" control. By selecting a "virtual" control law $\widehat{\chi}_3 = \alpha_2(\widehat{\chi}_1, \widehat{\chi}_2, \chi_1)$, we make the variable $z_1(t) = \chi_1(t)$ and $z_2(t) = \widehat{\chi}_2(t) - \alpha_1(\chi_1(t))$ tend to zero for $t \to +\infty$. Let us consider the following positive definite function:

$$V_2(z_1, z_2, e) = V_1(z_1, e) + \frac{1}{2}{z_2}^2 + W(e) > 0 \quad \text{for} \quad (z_1, z_2) \neq 0, \quad e \neq 0.$$

Below we use for convenience the variable $z_3 = \widehat{\chi}_3 - \alpha_2(\widehat{\chi}_1, \widehat{\chi}_2, \chi_1)$.

The time derivative of the function V_2 along the trajectories of system (46) is as follows:

$$\begin{gathered}\dot{V}_2|_{(46)} = \dot{V}_1|_{(46)} + z_2\dot{z}_2|_{(46)} + \dot{W}|_{(46)} \\ \leq -c_1z_1^2 + z_1z_2 - S_1 + z_2\dot{z}_2|_{(46)} + \dot{W}|_{(11)} \leq -c_1z_1^2 - S_1 \\ + z_2\Big(z_1 + \widehat{\chi}_3 + l_2(\widehat{\chi}_1 - \chi_1) + \varphi_2(\chi_1) - \frac{\partial\alpha_1}{\partial\chi_1}\Big(\widehat{\chi}_2 + \varphi_1(\chi_1) - e_2\Big)\Big) - \lambda e_2^2 \\ = -c_1z_1^2 - S_1 + z_2\Big(z_1 + z_3 + \alpha_2(\widehat{\chi}_1, \widehat{\chi}_2, \chi_1) + l_2(\widehat{\chi}_1 - \chi_1) + \varphi_2(\chi_1) - \frac{\partial\alpha_1}{\partial\chi_1}\Big(\widehat{\chi}_2 + \varphi_1(\chi_1) - e_2\Big)\Big) - \lambda e_2^2.\end{gathered}$$

By taking

$$\begin{gathered}\alpha_2(\widehat{\chi}_1, \widehat{\chi}_2, \chi_1) = -c_2z_2 - z_1 - l_2(\widehat{\chi}_1 - \chi_1) - \varphi_2(\chi_1) \\ + \frac{\partial\alpha_1}{\partial\chi_1}\Big(\widehat{\chi}_2 + \varphi_2(\chi_1)\Big) - d_2\left(\frac{\partial\alpha_1}{\partial\chi_1}\right)^2 z_2,\end{gathered}$$

where $c_2 > 0$ and $d_2 > 0$ are arbitrary positive constants, we obtain

$$\begin{aligned}\dot V_2|_{(46)} &\le -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 - \left(d_2\left(\frac{\partial\alpha_1}{\partial\chi_1}\right)^2 z_2^2 - \frac{\partial\alpha_1}{\partial\chi_1} z_2 e_2 + \lambda e_2^2\right) - S_1\\ &= -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 - S_1 - S_2,\end{aligned}$$

where

$$S_2 = \left(\sqrt{d_2}\,\frac{\partial\alpha_1}{\partial\chi_1} z_2 - \frac{1}{2\sqrt{d_2}} e_2\right)^2 + \left(\lambda - \frac{1}{4d_2}\right)e_2^2 \ge 0 \quad \text{for} \quad \lambda > \frac{1}{4d_2}.$$

Consequently, for an appropriate choice of the constants d_1 and d_2, the upper estimate of the time derivative of the function V_2 along the trajectories of system (46) consists of a negative definite part and the alternating-sign term $z_2 z_3$.

Step 3. By reasoning similarly, for the subsystem consisting of the three first equations of system (46) and represented as

$$\begin{aligned}\dot\chi_1 &= \alpha_1(\chi_1) + \varphi_1(\chi_1) - e_2 + (\widehat\chi_2 - \alpha_1(\chi_1)),\\ \dot{\widehat\chi}_2 &= \alpha_2(\widehat\chi_1, \widehat\chi_2, \chi_1) + l_2(\widehat\chi_1 - \chi_1) + \varphi_2(\chi_1) + (\widehat\chi_3 - \alpha_2(\widehat\chi_1, \widehat\chi_2, \chi_1)),\\ \dot{\widehat\chi}_3 &= \widehat\chi_4 + l_3(\widehat\chi_1 - \chi_1) + \varphi_3(\chi_1),\end{aligned}$$

we make use of the following positive definite function

$$V_3(z_1, z_2, z_3, e) = V_2(z_1, z_2, e) + \frac{1}{2} {z_3}^2 + W(e) > 0 \quad \text{for} \quad (z_1, z_2, z_3) \ne 0,\ e \ne 0$$

to construct the "virtual" control law $\widehat\chi_4 = \alpha_3(\widehat\chi_1, \widehat\chi_2, \widehat\chi_3, \chi_1)$. By taking the "virtual" control law $\widehat\chi_4 = \alpha_3(\widehat\chi_1, \widehat\chi_2, \widehat\chi_3, \chi_1)$, we make the variables $z_1(t) = \chi_1(t)$, $z_2(t) = \widehat\chi_2(t) - \alpha_1(\chi_1(t))$ and $z_3 = \widehat\chi_3(t) - \alpha_2(\widehat\chi_1(t), \widehat\chi_2(t), \chi_1(t))$ tend to zero for $t \to +\infty$. In what follows, we use for convenience the variable $z_4 = \widehat\chi_4 - \alpha_3(\widehat\chi_1, \widehat\chi_2, \widehat\chi_3, \chi_1)$.

The time derivative of the function V_3 along the trajectories of system (46) is represented as follows:

$$\begin{aligned}\dot V_3|_{(46)} &= \dot V_2|_{(46)} + z_3 \dot z_3|_{(46)} + \dot W|_{(46)}\\ &\le -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 - S_1 - S_2 + z_3 \dot z_3|_{(46)} + \dot W|_{(11)}\\ &\le -c_1 z_1^2 - c_2 z_2^2 - S_1 - S_2 + z_3\Bigg(z_2 + \widehat\chi_4 + l_3(\widehat\chi_1 - \chi_1) + \varphi_3(\chi_1)\\ &\quad - \frac{\partial\alpha_2}{\partial\chi_1}\Big(\widehat\chi_2 + \varphi_1(\chi_1) - e_2\Big) - \frac{\partial\alpha_2}{\partial\widehat\chi_1}\dot{\widehat\chi}_1 - \frac{\partial\alpha_2}{\partial\widehat\chi_2}\dot{\widehat\chi}_2\Bigg) - \lambda e_2^2\\ &= -c_1 z_1^2 - c_2 z_2^2 - S_1 - S_2 + z_3\Bigg(z_2 + z_4 + \alpha_3(\widehat\chi_1, \widehat\chi_2, \widehat\chi_3, \chi_1) + l_3(\widehat\chi_1 - \chi_1)\\ &\quad + \varphi_3(\chi_1) - \frac{\partial\alpha_2}{\partial\chi_1}\Big(\widehat\chi_2 + \varphi_1(\chi_1) - e_2\Big) - \frac{\partial\alpha_2}{\partial\widehat\chi_1}\dot{\widehat\chi}_1 - \frac{\partial\alpha_2}{\partial\widehat\chi_2}\dot{\widehat\chi}_2\Bigg) - \lambda e_2^2.\end{aligned}$$

By taking

$$\begin{aligned}\alpha_3(\widehat\chi_1, \widehat\chi_2, \widehat\chi_3, \chi_1) &= -c_3 z_3 - z_2 - l_3(\widehat\chi_1 - \chi_1) - \varphi_3(\chi_1)\\ &+ \frac{\partial\alpha_2}{\partial\chi_1}\Big(\widehat\chi_2 + \varphi_1(\chi_1)\Big) + \frac{\partial\alpha_2}{\partial\widehat\chi_1}\dot{\widehat\chi}_1 + \frac{\partial\alpha_2}{\partial\widehat\chi_2}\dot{\widehat\chi}_2 - d_3\left(\frac{\partial\alpha_2}{\partial\chi_1}\right)^2 z_3,\end{aligned}$$

where $c_3 > 0$ and $d_3 > 0$ are arbitrary positive constants, we obtain

$$\dot{V}_3|_{(46)} \le -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4 - \left(d_3 \left(\frac{\partial \alpha_2}{\partial \chi_1} \right)^2 z_3^2 - \frac{\partial \alpha_2}{\partial \chi_1} z_3 e_2 + \lambda e_2^2 \right) - S_1 - S_2$$
$$= -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4 - S_1 - S_2 - S_3,$$

where

$$S_3 = \left(\sqrt{d_3}\, \frac{\partial \alpha_2}{\partial \chi_1} z_3 - \frac{1}{2\sqrt{d_3}} e_2 \right)^2 + \left(\lambda - \frac{1}{4d_3} \right) e_2^2 \ge 0 \quad \text{for} \quad \lambda > \frac{1}{4d_3}.$$

Consequently, for an appropriate choice of the constants d_1, d_2, and d_3, the upper estimate of the time derivative of the function V_3 along the trajectories of system (46) consists of the negative definite part and the alternating-sign term $z_3 z_4$.

Now, Steps 4–$(n-1)$ of the algorithm to construct the stabilizing control are similar to Step 3 and lie in constructing the corresponding "virtual" control laws

$$\widehat{\chi}_i = \alpha_{i-1}(\widehat{\chi}_1, \ldots, \widehat{\chi}_{i-1}, \chi_1), \quad i = \overline{5, n},$$

using the positive definite functions $i = \overline{4, n-1}$,

$$V_i(z_1, \ldots, z_i, e) = V_{i-1}(z_1, \ldots, z_{i-1}, e) + \frac{1}{2} {z_i}^2 + W(e) > 0 \quad \text{for} \quad (z_1, \ldots, z_i) \ne 0,\ e \ne 0.$$

Step n. At the concluding stage, we select a real control $u = \alpha_n(\widehat{\chi}, \chi_1)$ to drive to zero the variables

$$\begin{gathered} z_1(t) = \chi_1(t),\ z_2(t) = \widehat{\chi}_2(t) - \alpha_1(\chi_1(t)), \\ z_3(t) = \widehat{\chi}_3(t) - \alpha_2(\widehat{\chi}_1(t), \widehat{\chi}_2(t), \chi_1(t)), \quad \ldots, \\ z_n(t) = \widehat{\chi}_n(t) - \alpha_{n-1}(\widehat{\chi}_1(t), \ldots, \widehat{\chi}_{n-1}(t), \chi_1(t)) \end{gathered} \tag{A.10}$$

for $t \to +\infty$. Let us consider the following positive definite function:

$$V_n(z_1, \ldots, z_n, e) = V_{n-1}(z_1, \ldots, z_{n-1}, e) + \frac{1}{2} {z_n}^2 + W(e) > 0 \ \text{ for } \ (z_1, \ldots, z_n) \ne 0,\ e \ne 0.$$

Its time derivative along the trajectories of system (46) is as follows:

$$\dot{V}_n|_{(46)} = \dot{V}_{n-1}|_{(46)} + z_n \dot{z}_n|_{(46)} + \dot{W}|_{(46)}$$
$$\le -\sum_{i=1}^{n-1} c_i z_i^2 - \sum_{i=1}^{n-1} S_i + z_n z_{n-1} + z_n \dot{z}_n|_{(46)} + \dot{W}|_{(11)}$$
$$\le -\sum_{i=1}^{n-1} c_i z_i^2 - \sum_{i=1}^{n-1} S_i + z_n \Bigg(z_{n-1} + l_n(\widehat{\chi}_1 - \chi_1) + \beta(\chi_1) u + \varphi_n(\chi_1)$$
$$- \frac{\partial \alpha_{n-1}}{\partial \chi_1} \Big(\widehat{\chi}_2 + \varphi_1(\chi_1) - e_2 \Big) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \widehat{\chi}_i} \dot{\widehat{\chi}}_i \Bigg) - \lambda \sum_{i=1}^{n} e_i^2.$$

Then, if one takes the control law

$$u = \alpha_n(\widehat{\chi}, \chi_1) = \frac{1}{\beta(\chi_1)} \Bigg(-c_n z_n - z_{n-1} - l_n(\widehat{\chi}_1 - \chi_1) - \varphi_n(\chi_1)$$
$$+ \frac{\partial \alpha_{n-1}}{\partial \chi_1} \Big(\widehat{\chi}_2 + \varphi_1(\chi_1) \Big) + \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \widehat{\chi}_i} \dot{\widehat{\chi}}_i - d_n \left(\frac{\partial \alpha_{n-1}}{\partial \chi_1} \right)^2 z_n \Bigg), \tag{A.11}$$

where $c_n > 0$ and $d_n > 0$ are arbitrary positive constants, the following estimate is true for the time derivative of the function V_n along the trajectories of the closed-loop system (46):

$$\begin{aligned}\dot{V}_n|_{(46)} &\le -\sum_{i=1}^{n} c_i z_i^2 - \sum_{i=1}^{n} S_i - \lambda e_1^2 - \lambda \sum_{i=3}^{n} e_i^2 \\ &= -\sum_{i=1}^{n} c_i z_i^2 - \left(\sqrt{d_1} z_1 + \frac{1}{2\sqrt{d_1}} e_2\right)^2 \\ -\sum_{i=2}^{n}\left(\sqrt{d_i}\,\frac{\partial \alpha_{i-1}}{\partial \chi_1} z_i - \frac{1}{2\sqrt{d_i}} e_2\right)^2 &- \sum_{i=1}^{n}\left(\lambda - \frac{1}{4d_i}\right) e_2^2 - \lambda e_1^2 - \lambda \sum_{i=3}^{n} e_i^2.\end{aligned} \tag{A.12}$$

If the constants d_i, $i = \overline{1, n}$, are selected so that inequality (48) is satisfied, then the estimate

$$\dot{V}_n|_{(46)} \le -\sum_{i=1}^{n} c_i z_i^2 - \widetilde{\lambda} \sum_{i=1}^{n} e_i^2, \tag{A.13}$$

where $\widetilde{\lambda} = \min\left\{\lambda, \sum\limits_{i=1}^{n}\left(\lambda - \dfrac{1}{4d_i}\right)\right\} > 0$, is valid.

Relations (A.10) and $e = e$ define the map which is a diffeomorphism of the spaces $R^{2n} = \{\chi_1, \widehat{\chi}_2, \ldots, \widehat{\chi}_n, e\}$ and $R^{2n} = \{z_1, \ldots, z_n, e\}$ and represents a globally defined change of variables. In terms of the variables z_i, $i = \overline{1, n}$, and e, system (46) closed by control (A.11) takes the form

$$\begin{aligned}\dot{z}_1 &= -c_1 z_1 + z_2 - d_1 z_1 - e_2, \\ \dot{z}_2 &= -c_2 z_2 - z_1 + z_3 - d_2\left(\frac{\partial \alpha_1}{\partial \chi_1}\right)^2 z_2 + \frac{\partial \alpha_1}{\partial \chi_1} e_2, \\ &\vdots \\ \dot{z}_{n-1} &= -c_{n-1} z_{n-1} - z_{n-2} + z_n - d_{n-1}\left(\frac{\partial \alpha_{n-2}}{\partial \chi_1}\right)^2 z_{n-1} + \frac{\partial \alpha_{n-2}}{\partial \chi_1} e_2, \\ \dot{z}_n &= -c_n z_n - z_{n-1} - d_n\left(\frac{\partial \alpha_{n-1}}{\partial \chi_1}\right)^2 z_n + \frac{\partial \alpha_{n-1}}{\partial \chi_1} e_2, \\ \dot{e} &= (A + LC)e,\end{aligned} \tag{A.14}$$

where $z = (z_1, \ldots, z_n)^{\mathrm{T}} \in R^n$ and $e \in R^n$ make up the system state vector. We note that, by virtue of (A.10), system (A.14) is stationary because its right-hand side depends only on z and e.

The time derivative of the function V_n along the trajectories of system (A.14) for all $z \in R^n$ and $e \in R^n$ satisfies (A.12) and, provided that inequality (48) is satisfied, also estimate (A.13). Consequently, if inequality (48) is satisfied, then V_n is a Lyapunov function for system (A.14), and according to [175], the equilibrium $z = 0$, $e = 0$ of system (A.14) is globally exponentially stable. Since $\alpha_i(0) = 0$, $i = \overline{1, n-1}$, it now follows from (A.10) that the equilibrium $\widehat{\chi} = 0$, $e = 0$ of system (46) closed by control (A.11) is globally asymptotically stable, which by virtue of $\chi = \widehat{\chi} - e$ also implies the global asymptotic stability of the equilibrium $\chi = 0$, $e = 0$ of system (45) under control (A.11).

Part of the results presented in this paper is compiled as Tables 1 and 2 of the classes of nonlinear systems admitting construction of the asymptotic observers, conditions for and type of stability of the system describing dynamics of the error of state estimation of the original system by the observer, conditions for validity and type (in the case of local, global asymptotic stabilization or asymptotic stabilization in the large) of the separation principle.

Table 1. Observers of nonlinear systems

System class	Observer	Type of stability	Sufficient stability conditions
$\dot{x} = Ax + \varrho(y,u)$, $y = Cx$, $x \in R^n$, $u \in R^m$, $A \in R^{n\times n}$, $C \in R^{p\times n}$, the pair (A, C) is detectable, $\varrho(y,u)$ continuous	$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + \varrho(y,u)$, $L \in R^{n\times p}$	Global exponential	Hurwitz stability of the matrix $A + LC$
$\dot{x} = Ax + \widetilde{f}(x,u) + \varrho(y,u)$, $y = Cx$, $x \in R^n$, $u \in R^m$, $A \in R^{n\times n}$, $C \in R^{p\times n}$, the (A, C) is detectable, $\varrho(y,u)$ continuous, $\widetilde{f}(x,u)$ is continuous and globally Lipschitzian in x uniformly in u	$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + \widetilde{f}(\widehat{x},u) + \varrho(y,u)$, $L \in R^{n\times p}$	Global exponential	Theorem 3.1
$\dot{x}_1 = x_2 + b_1(x_1,u) + \varrho_1(y,u)$, $\dot{x}_2 = x_3 + b_2(x_1,x_2,u) + \varrho_2(y,u)$, $\vdots$ $\dot{x}_{n-1} = x_n + b_{n-1}(x_1,\dots,x_{n-1},u) + \varrho_{n-1}(y,u)$, $\dot{x}_n = a_n(x) + b_n(x,u) + \varrho_n(y,u)$, $y = x_1$, $u \in R^m$, $x = (x_1,\dots,x_n)^{\mathrm{T}} \in R^n$, $a_n(x)$ globally Lipschitzian, $b_i(x,u)$, $i = \overline{1,n}$, continuous and globally Lipschitzian in x uniformly in u, $\varrho_i(y,u)$, $i = \overline{1,n}$, continuous	$\dot{\widehat{x}}_1 = \widehat{x}_2 + l_1(\widehat{x}_1 - x_1) + b_1(\widehat{x}_1,u) + \varrho_1(y,u)$, $\dot{\widehat{x}}_2 = \widehat{x}_3 + l_2(\widehat{x}_1 - x_1) + b_2(\widehat{x}_1,\widehat{x}_2,u) + \varrho_2(y,u)$, $\vdots$ $\dot{\widehat{x}}_{n-1} = \widehat{x}_n + l_{n-1}(\widehat{x}_1 - x_1) + b_{n-1}(\widehat{x}_1,\dots,\widehat{x}_{n-1},u) + \varrho_{n-1}(y,u)$, $\dot{\widehat{x}}_n = a_n(\widehat{x}) + l_n(\widehat{x}_1 - x_1) + b_n(\widehat{x},u) + \varrho_n(y,u)$, $L = (l_1,\dots,l_n)^{\mathrm{T}} \in R^n$	Global exponential	Theorems 3.2, 3.3
$\dot{x} = Ax + G\psi(Hx) + \varrho(y,u)$, $y = Cx$, $x \in R^n$, $u \in R^m$, $A \in R^{n\times n}$, $G \in R^{n\times r}$, $H \in R^{r\times n}$, $C \in R^{p\times n}$, the pair (A, C) is detectable, $\varrho(y,u)$ and $\psi(Hx)$ are continuous, $\psi(Hx) = \Big(\psi_1\big(\sum_{j=1}^n H_{1j}x_j\big),\dots,\psi_r\big(\sum_{j=1}^n H_{rj}x_j\big)\Big)^{\mathrm{T}}$, $a \le \dfrac{\psi_i(z_1) - \psi_i(z_2)}{z_1 - z_2} \le b$, $\forall z_1, z_2 \in R$, $z_1 \ne z_2$, $i = \overline{1,r}$	$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + G\psi\big(H\widehat{x} + K(C\widehat{x} - y)\big) + \varrho(y,u)$, $L \in R^{n\times p}$, $K \in R^{r\times p}$	Global exponential	Theorem 3.4
$\dot{x} = f(x,u)$, $y = Cx$, $x \in R^n$, $u \in R^m$, $C \in R^{p\times n}$, $f(x,u)$ continuously differentiable	$\dot{\widehat{x}} = f(\widehat{x},u) + L(u)(C\widehat{x} - y)$, $L(u)$ continuous	Global exponential	Theorem 3.5

Table 2. Satisfaction of the separation principle

System class	Observer	Type separation of principle	Sufficient validity conditions
$\dot{x} = f(x,u)$, $y = h(x)$, $x \in R^n$, $u \in R^m$, $y \in R^p$, $f(x,u)$ and $h(x)$ are continuous, $f(0,0) = 0$, $h(0) = 0$	$\dot{\widehat{x}} = g(\widehat{x}, h(x), u)$, $g(\cdot)$ is continuous, $g(0,0,0) = 0$, asymptotic, operates locally	In the case of local asymptotic stabilization	Theorem 4.1, generalization of Theorem 4.1
	$\dot{\widehat{x}} = g(\widehat{x}, h(x), u)$, $g(\cdot)$ is continuous, $g(0,0,0) = 0$, asymptotic, operates globally	In the case of global asymptotic stabilization	Theorems 4.3–4.5
$\dot{x} = Ax + B\phi(x,z,u)$, $\dot{z} = \psi(x,z,u)$, $y = Cx$, $\zeta = q(x,z)$, $x \in X \subseteq R^r$, $z \in Z \subseteq R^l$, $y \in Y \subseteq R^p$, $\zeta \in R^s$, $u \in U \subseteq R^m$, maps $\phi\colon X \times Z \times U \to R^p$ and $\psi\colon X \times Z \times U \to R^l$ are locally Lipschitzian, $\phi(0,0,0) = 0$, $\psi(0,0,0) = 0$, $q(0,0) = 0$; $A \in R^{r \times r}$, $B \in R^{r \times p}$, $C \in R^{p \times r}$, $A = \operatorname{diag}(A^1, \ldots, A^p)$, $B = \operatorname{diag}(B^1, \ldots, B^p)$, $C = \operatorname{diag}(C^1, \ldots, C^p)$, $A^k = (a^k_{ij})$, $i = \overline{1, r_k}$, $j = \overline{1, r_k}$, $k = \overline{1, p}$, are square matrices of orders r_k with the elements $a^k_{ij} = 1$, if $j - i = 1$, and $a^k_{ij} = 0$, if $j - i \neq 1$; $B^k = (0, \ldots, 0, 1)^{\mathrm{T}}$, $k = \overline{1,p}$, are the column vectors of height r_k; $C^k = (1, 0, \ldots, 0)$, $k = \overline{1,p}$, are the row matrices of length r_k, $r = r_1 + \ldots + r_p$	$\dot{\widehat{x}} = A\widehat{x} + L(C\widehat{x} - y) + B\phi_0(\widehat{x}, z, u)$, $\phi_0(x,z,u)$ is locally Lipschitzian and bounded in x, $\phi_0(0,0,0) = 0$; $L \in R^{r \times p}$, $L = \operatorname{diag}(L^1, \ldots, L^p)$, $L^i = (\theta k^i_1, \theta^2 k^i_2, \ldots, \theta^{r_i} k^i_{r_i})^{\mathrm{T}}$, $i = \overline{1,p}$; $\theta > 1$, k^i_j, $j = \overline{1, r_i}$, $i = \overline{1,p}$; are the components of the vectors $K^i = (k^i_1, k^i_2, \ldots, k^i_{r_i})^{\mathrm{T}}$, $i = \overline{1,p}$, such that the matrices $A^i + K^i C^i$, $i = \overline{1,p}$, are Hurwitzian, asymptotic with high gains, operates in the large	In the case of asymptotic in the large	Theorem 4.2
$\dot{x} = f(x,u)$, $y = h(x)$, $x \in R^n$, $u \in R$, $y \in R$, $f(x,u)$ and $h(x)$ are smooth, $f(0,0) = 0$, $h(0) = 0$	Asymptotic with high gains, operates in the large (see Section 4.2)	In the case of asymptotic stabilization in the large	Global asymptotic stabilizability of the system by a smooth state feedback and uniform system observability under an arbitrary control

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